Topology and Neuroscience:
Data analysis

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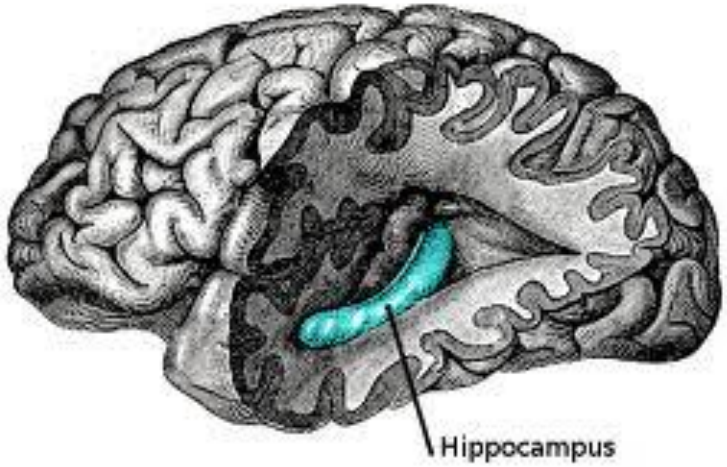
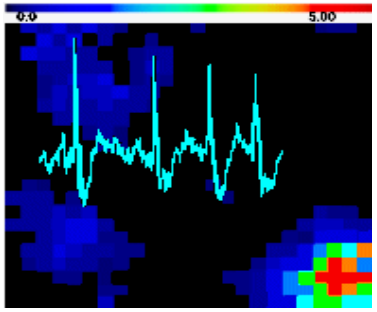
Hippocampus and space coding

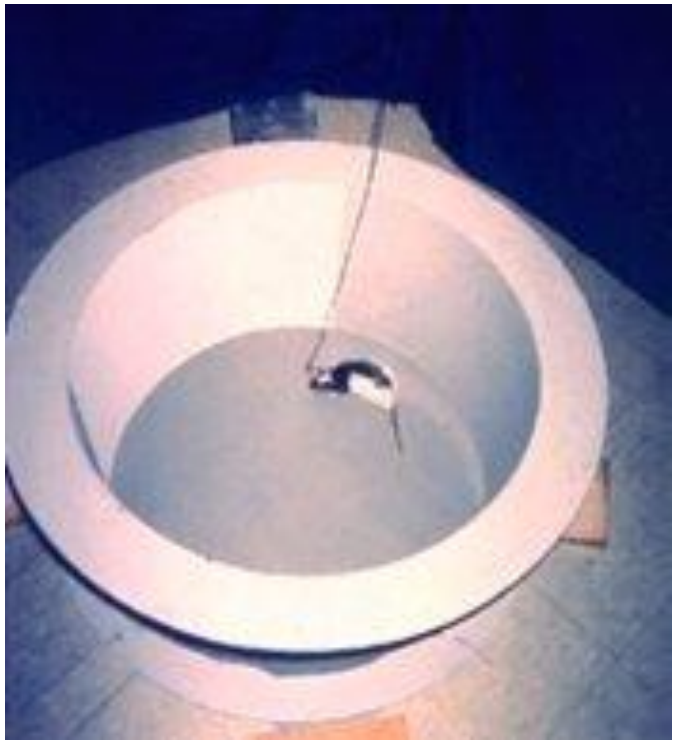


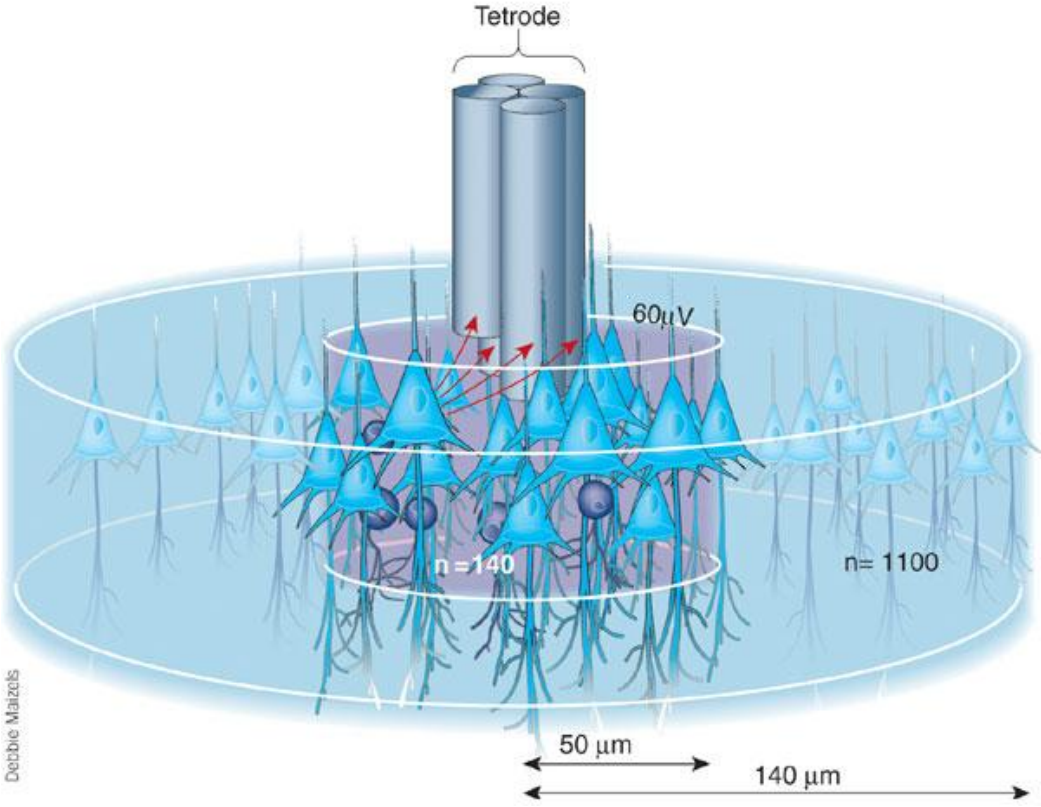
- Forming new memories

- Spatial behavior, spatial memory
  - navigation, navigation planning
  - imagining spatial scenes

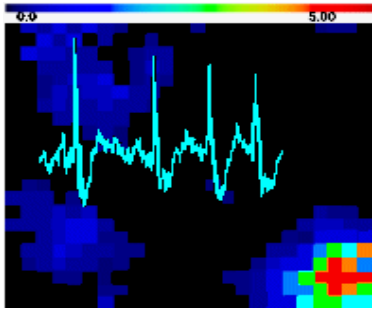
Electrophysiology of spatial coding

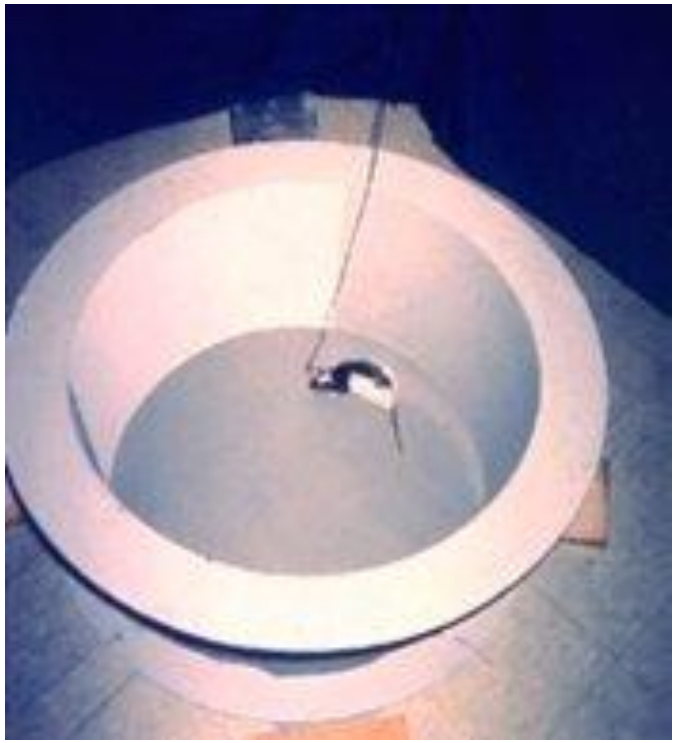


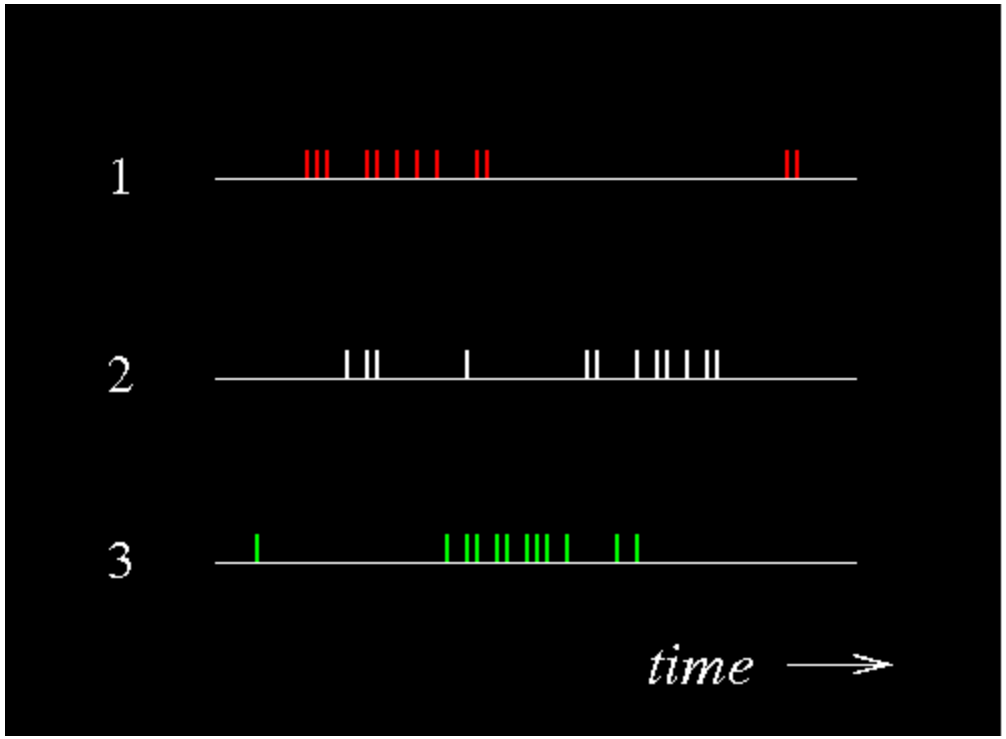




Electrophysiology of spatial coding

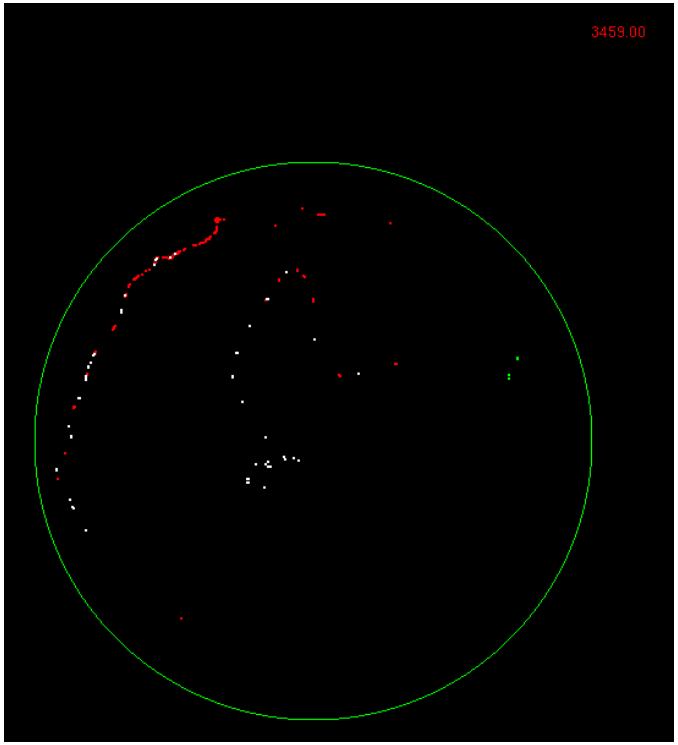


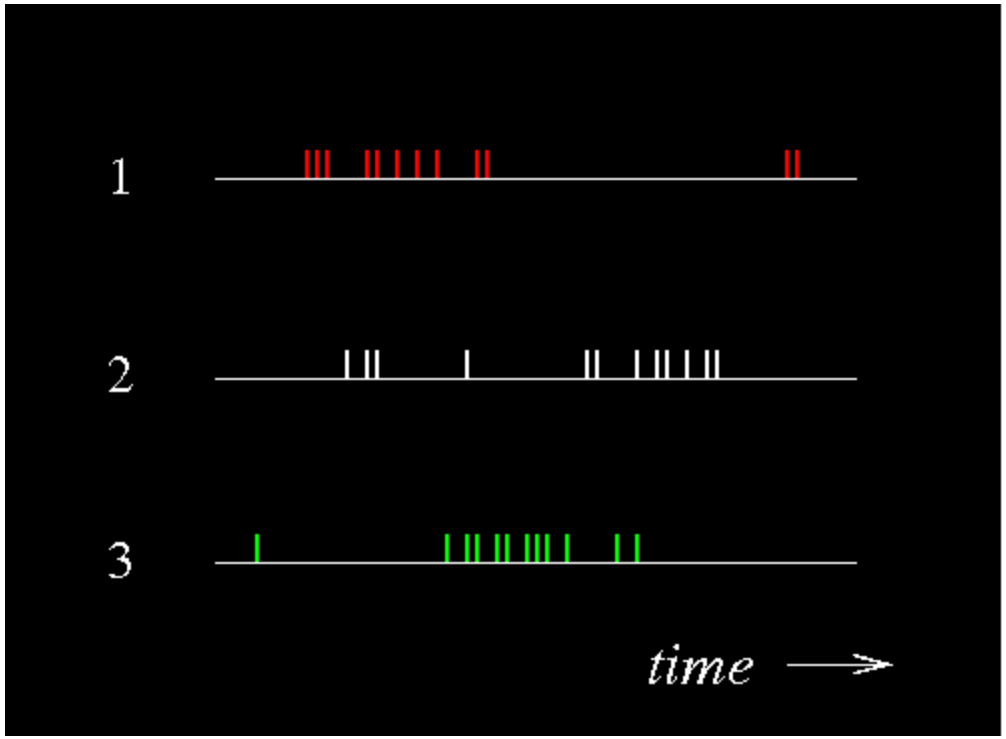




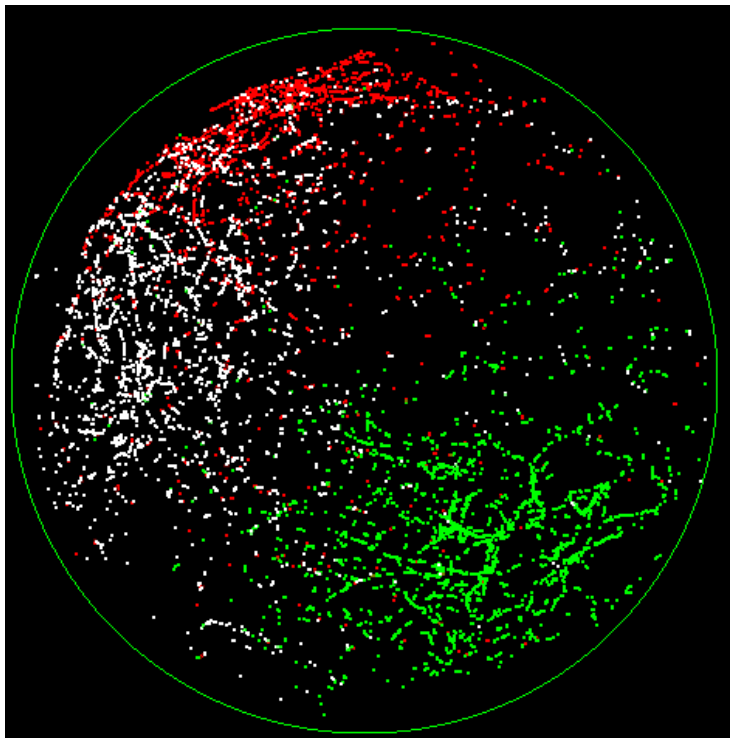
Electrophysiology of spatial coding

Firing activity is correlated with spatial location





Hippocampal spatial map

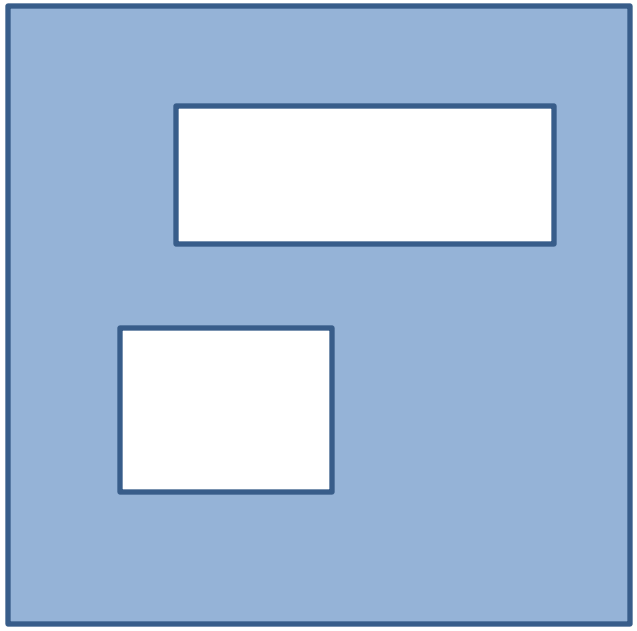


~ 4,000 cells are active simultaneously on each side

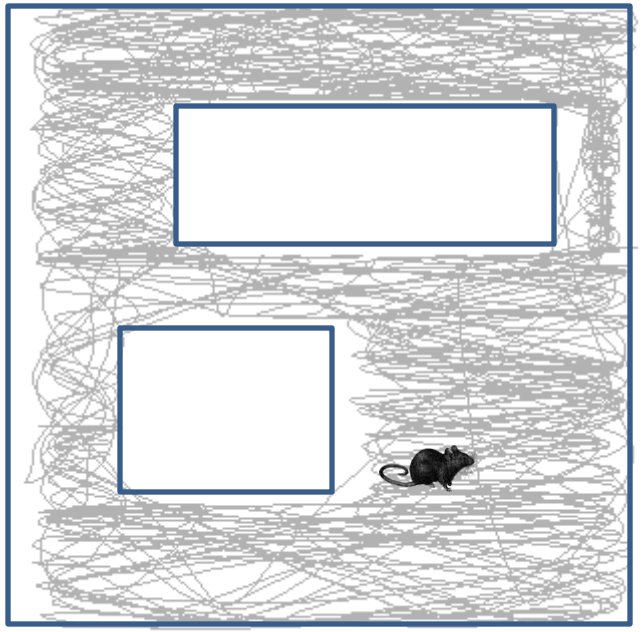
~400,000 cells total on each side

The ensemble of place cells forms a map of the environment

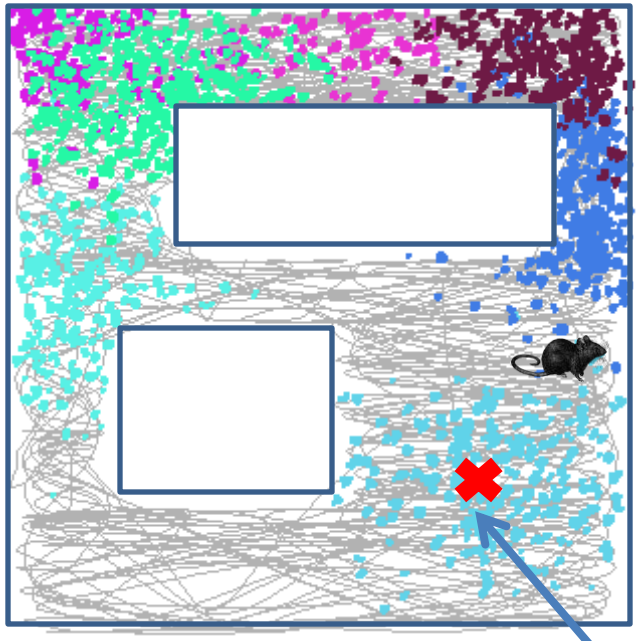
A space with 2 holes

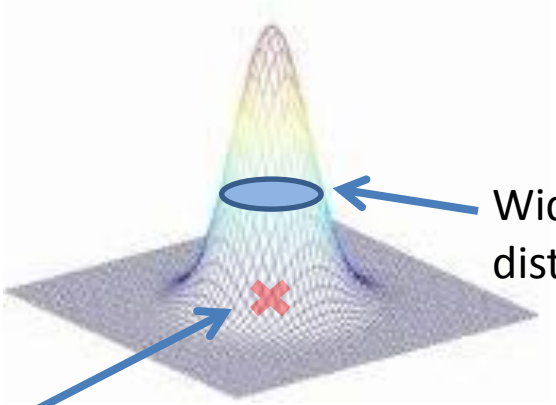


Environment with 2 holes + trajectory



Examples of simulated place fields





Maximal firing rate, F

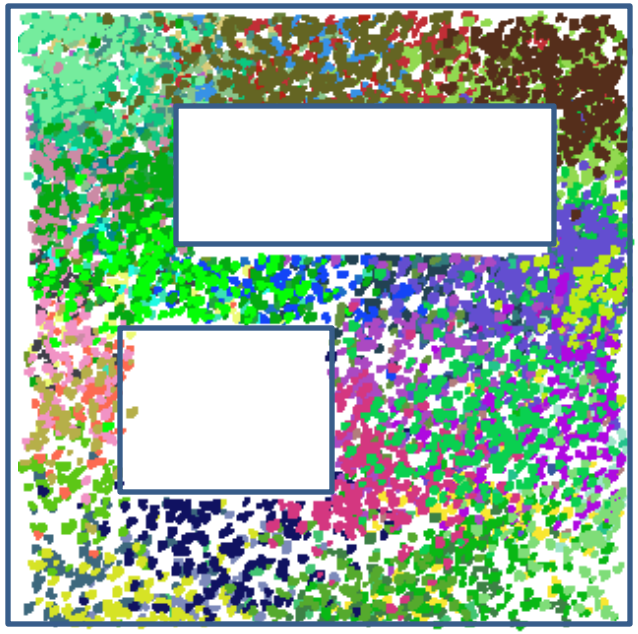
Width of the distribution, s

Place field center, $\vec{r}_0$

Firing probability distribution

$$f\left(\vec{r}\right) = Fe^{-\frac{\left(\vec{r}-\vec{r}_0\right)^2}{s^2}}$$

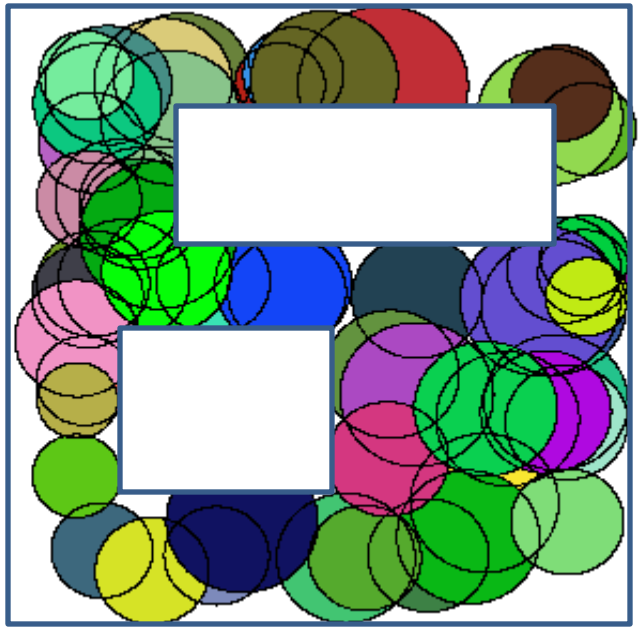
A place field map of a 2 hole environment

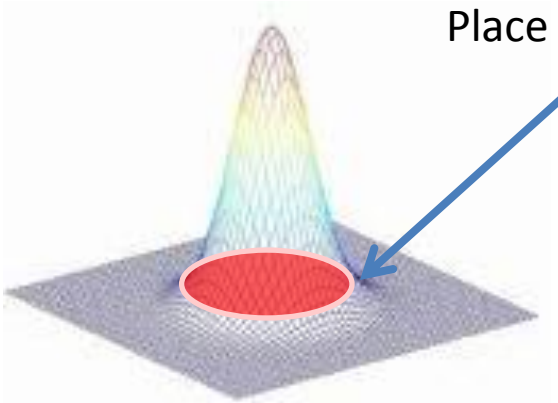


An ensemble of N place cells forms a map of the environment

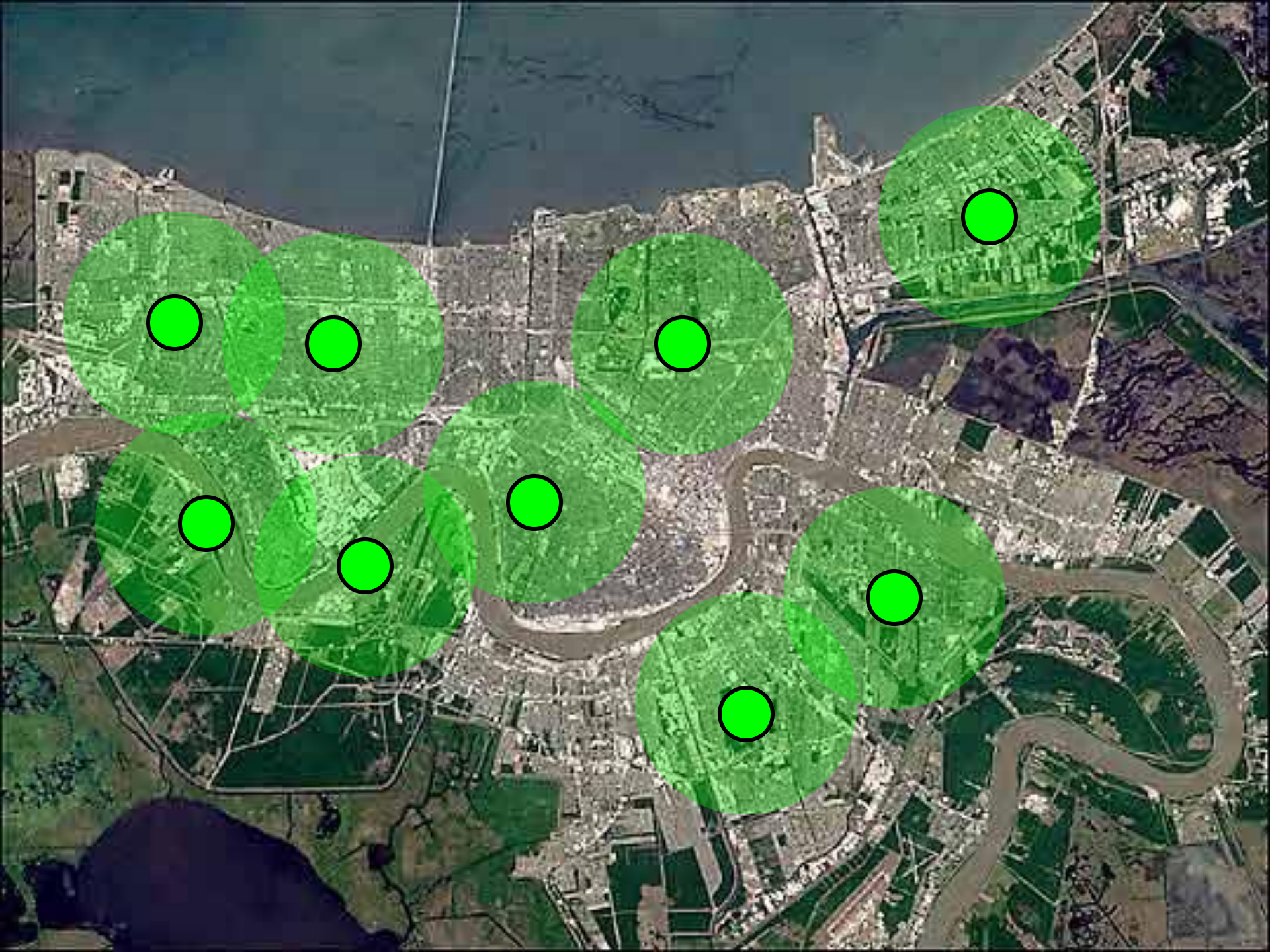
Information transmitted by the place cells to the downstream neurons

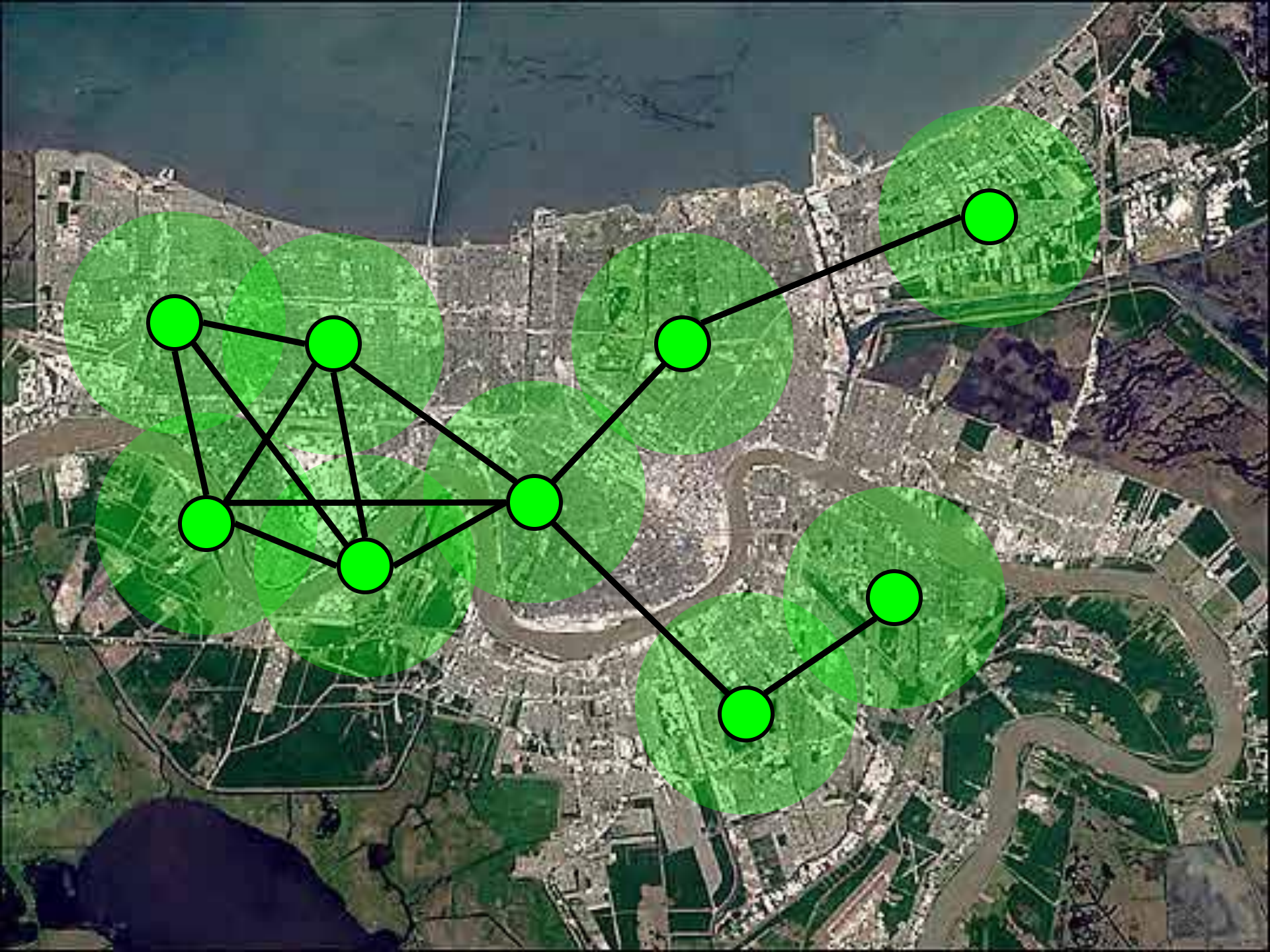
A schematic structure of the place field map



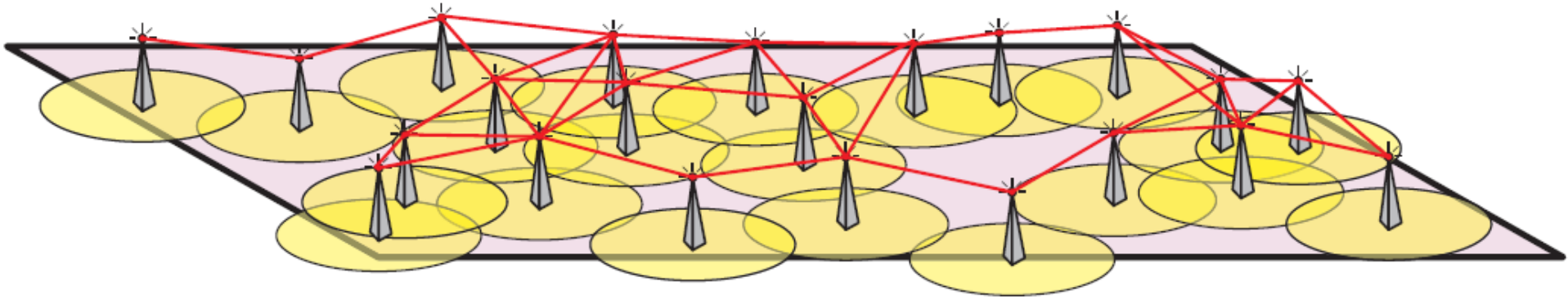


Information transmitted by the place cells to the downstream neurons



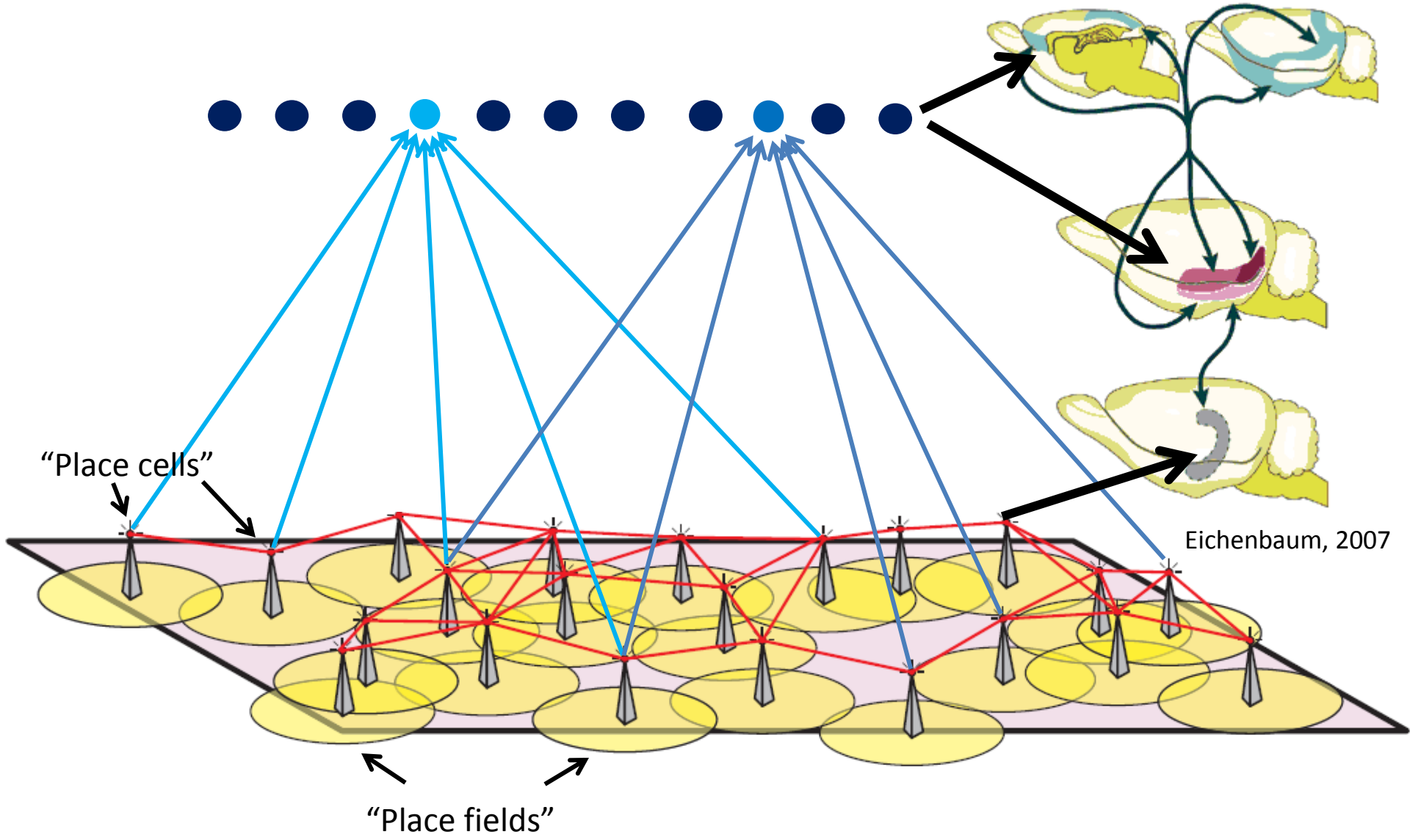


Topology from sensor networks

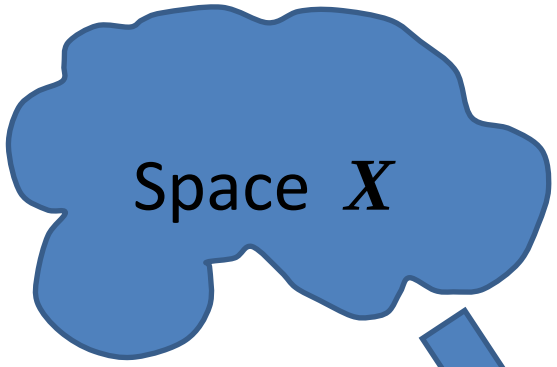


V. de Silva, Homological sensor networks, (2007)

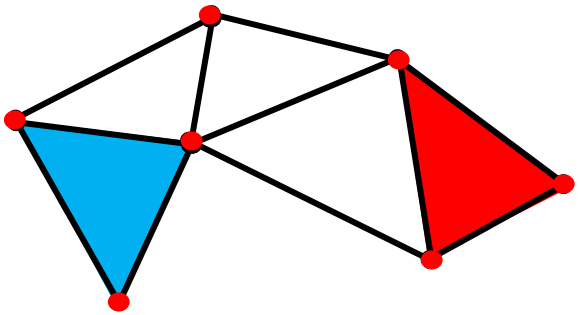
Topology from neural networks

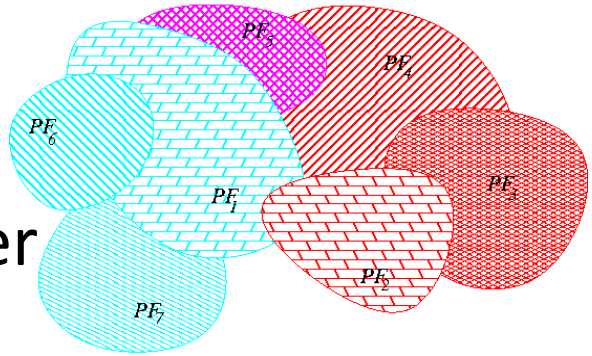


Topological analysis - Čech complex



Simplicial complex K





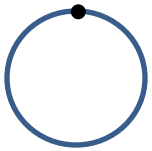
Space cover

Homologies $H_q(K) \sim H_q(X)$

Betti index – base cycles in every dimension

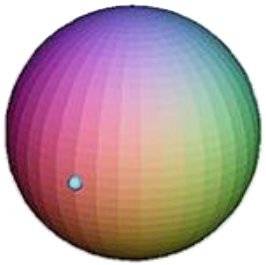
"Topological barcode"

Circle



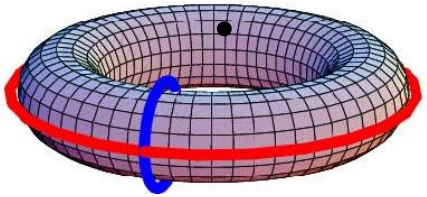
(1, 1, 0, 0,…)

Sphere



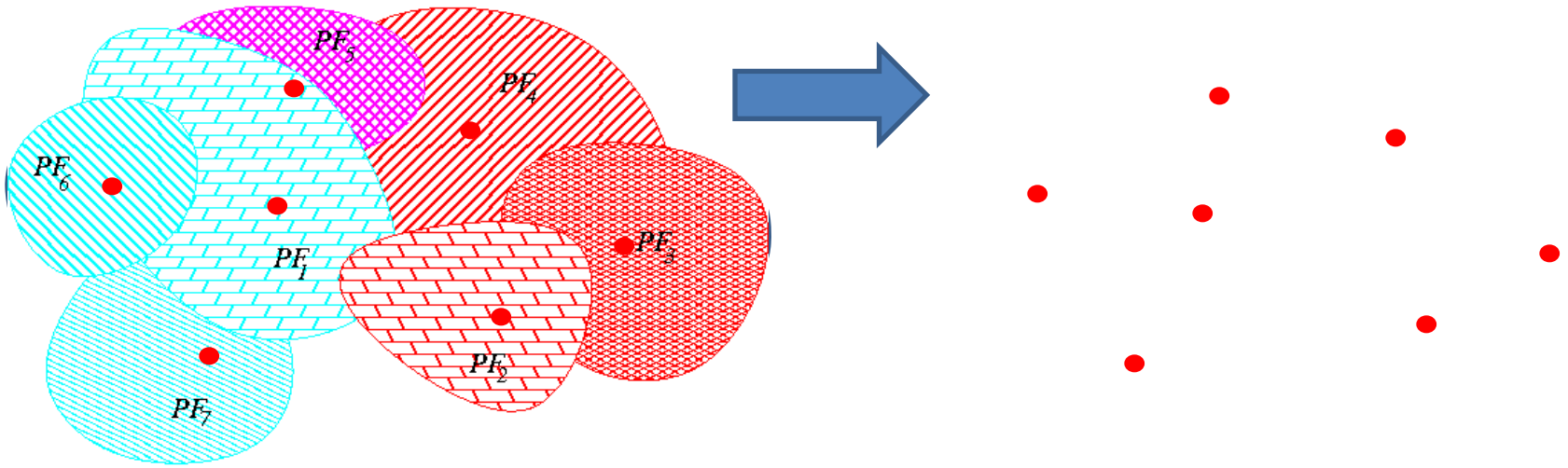
(1, 0 , 1, 0,…)

Torus



(1, 2 , 1, 0,…)

Čech complex - review



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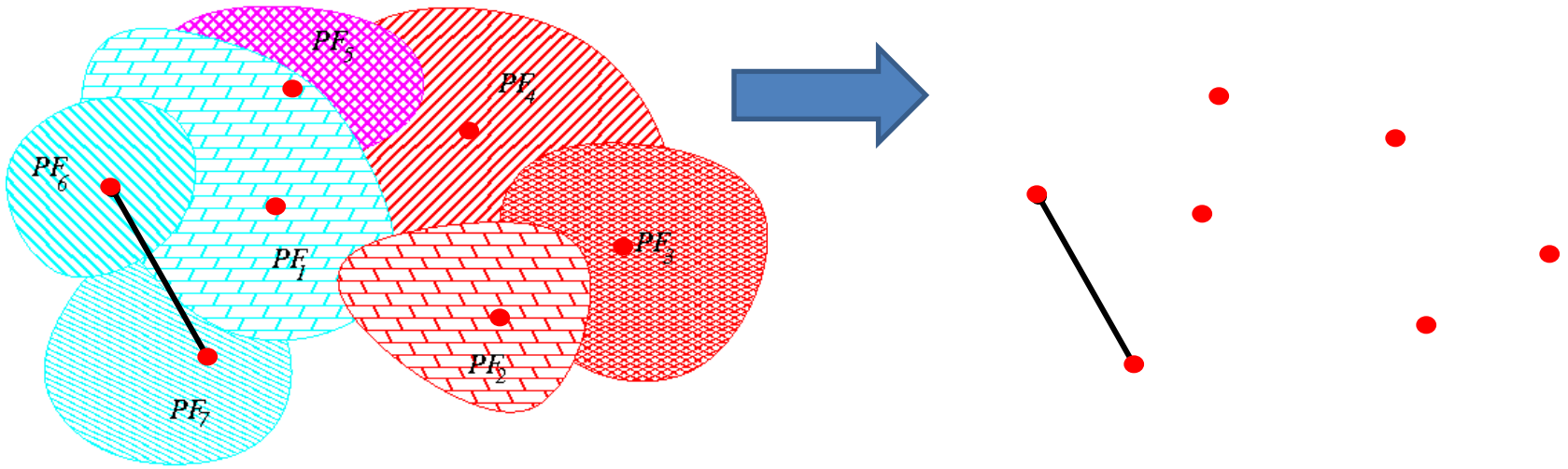

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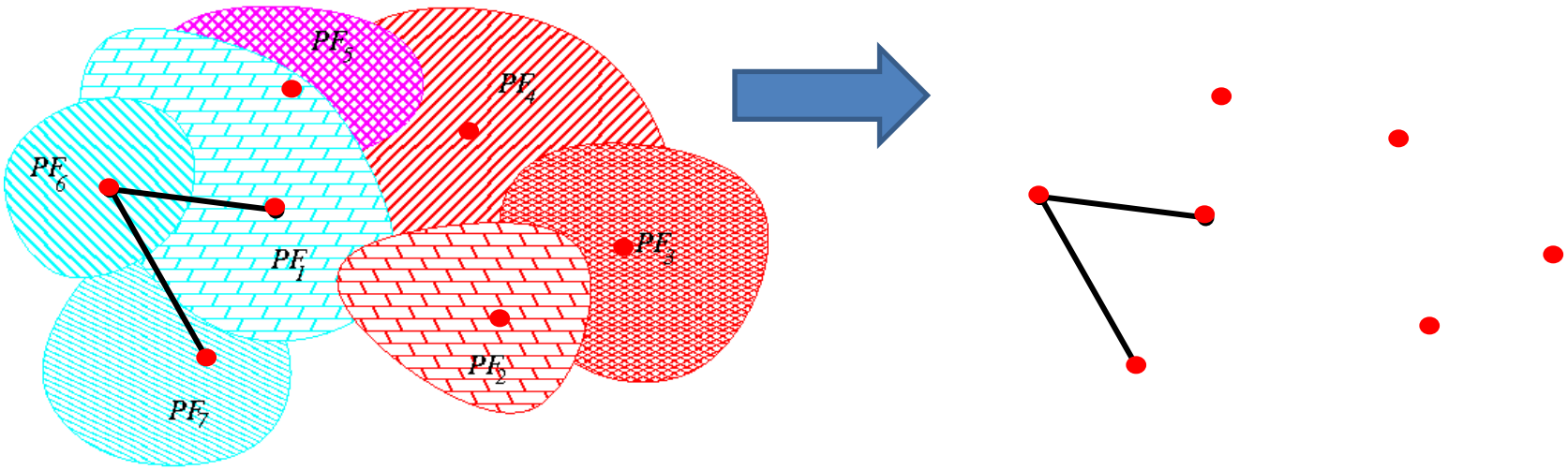

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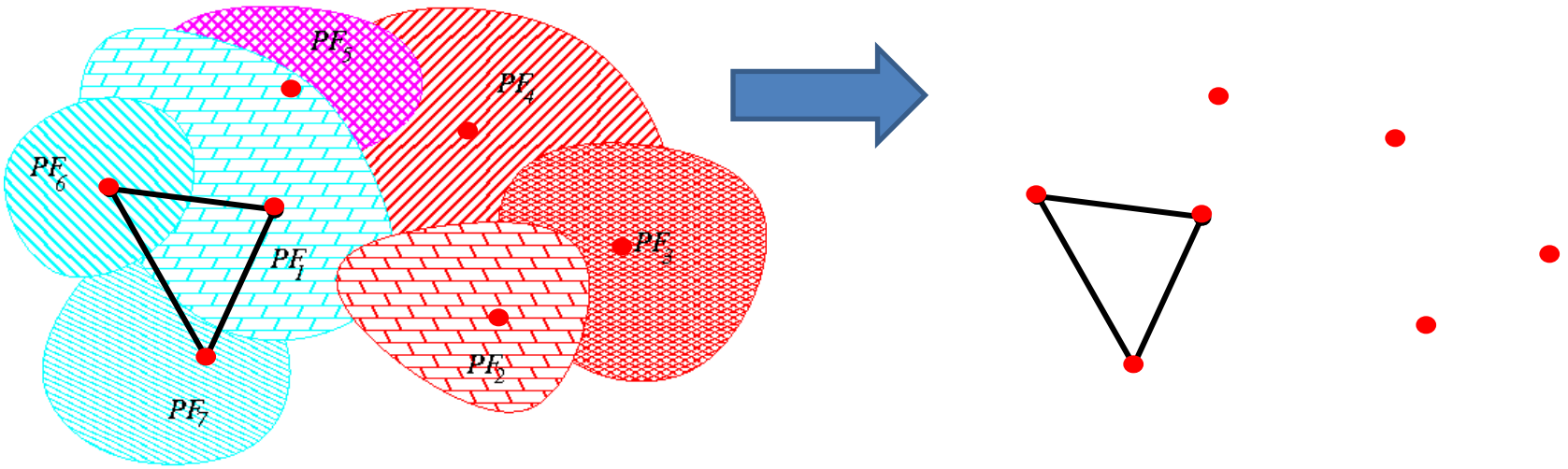

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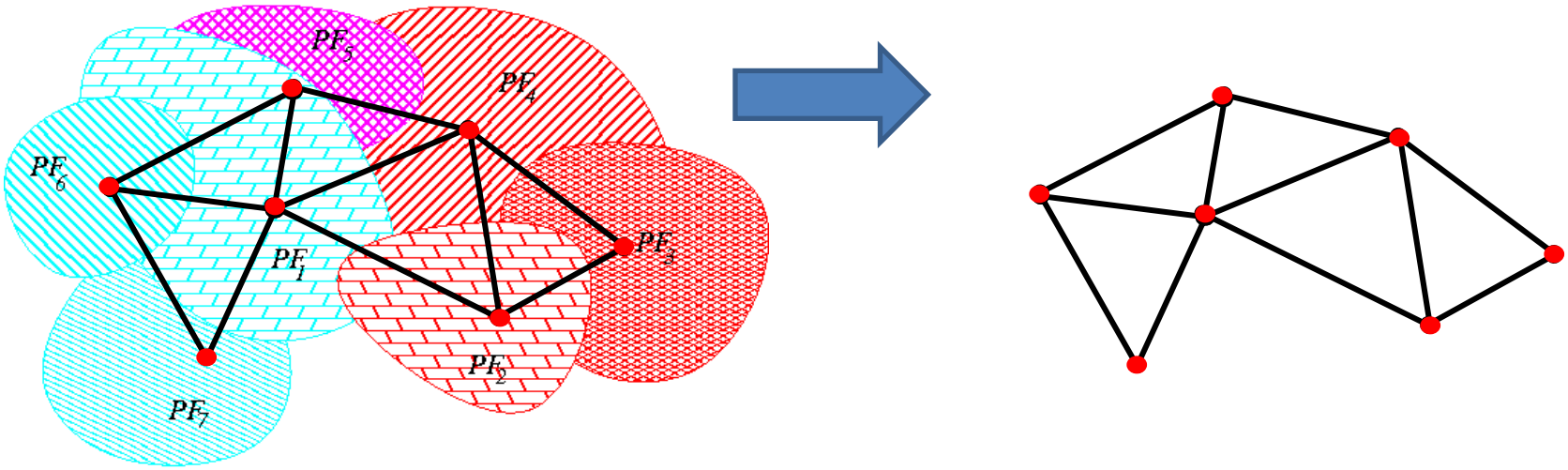

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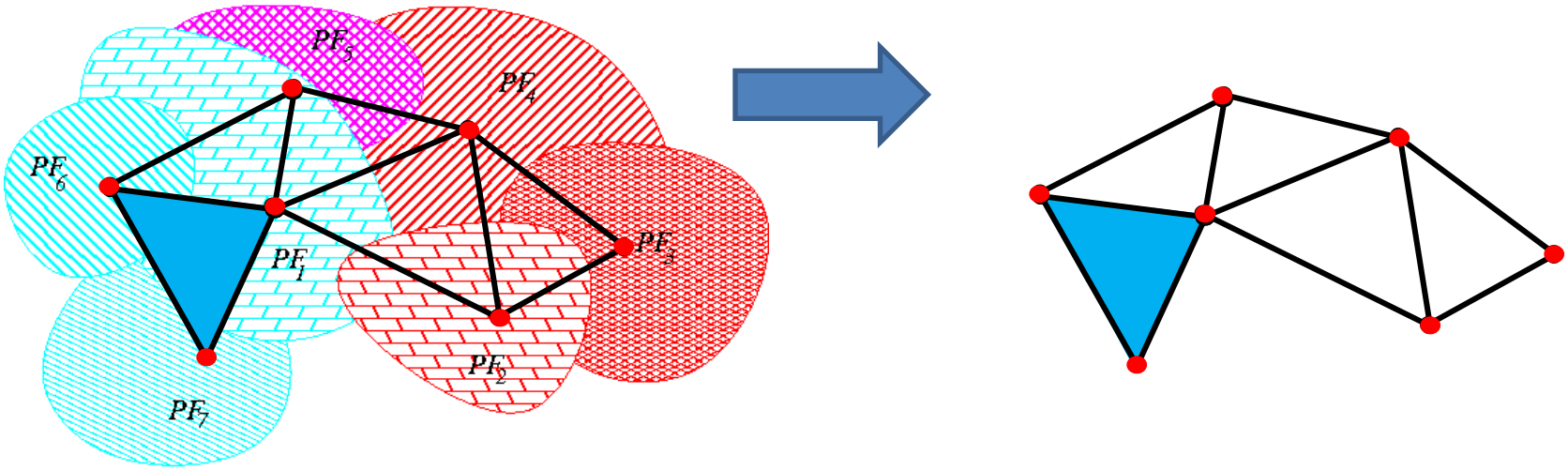

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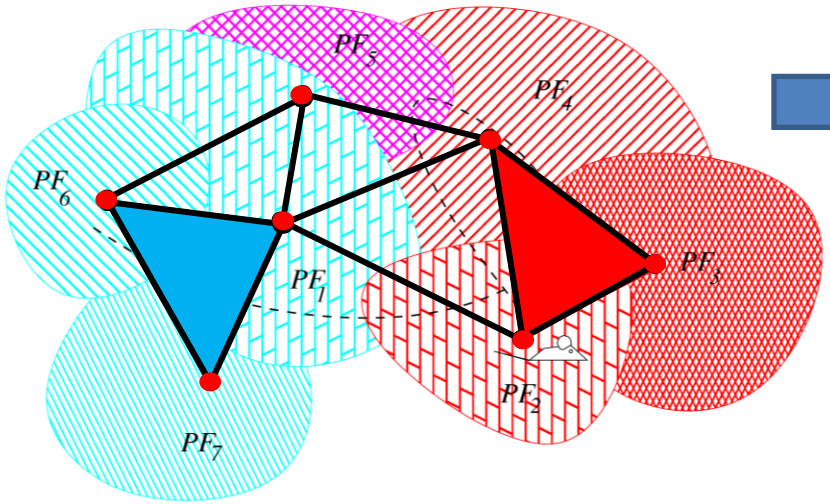
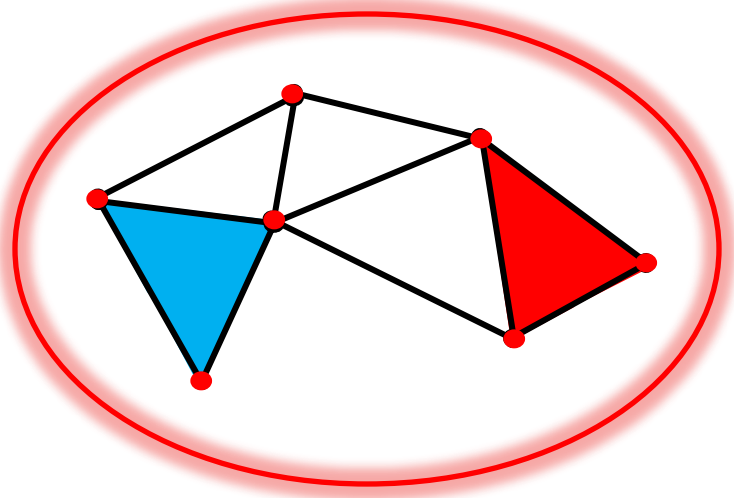
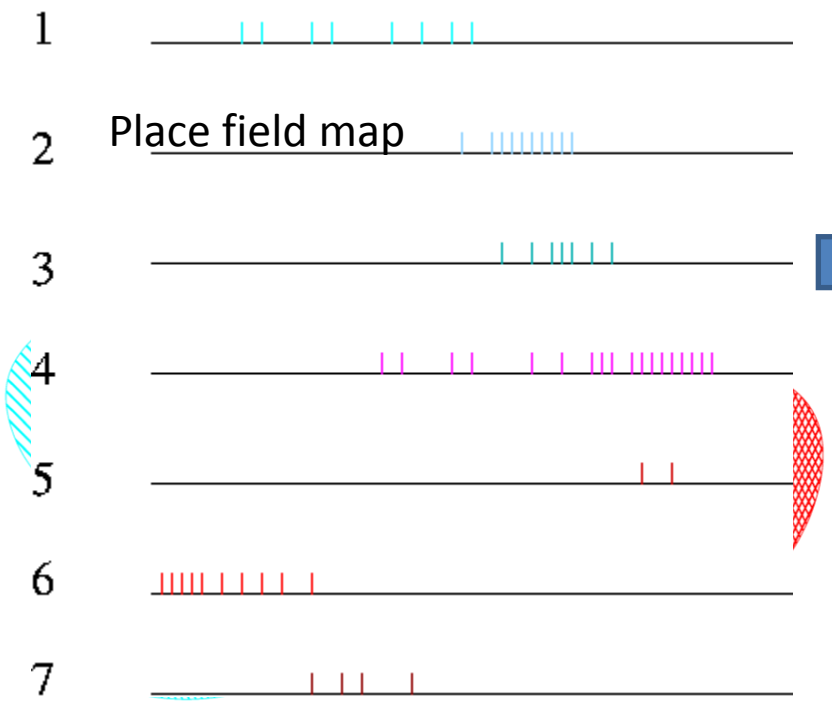


Simplicial complex



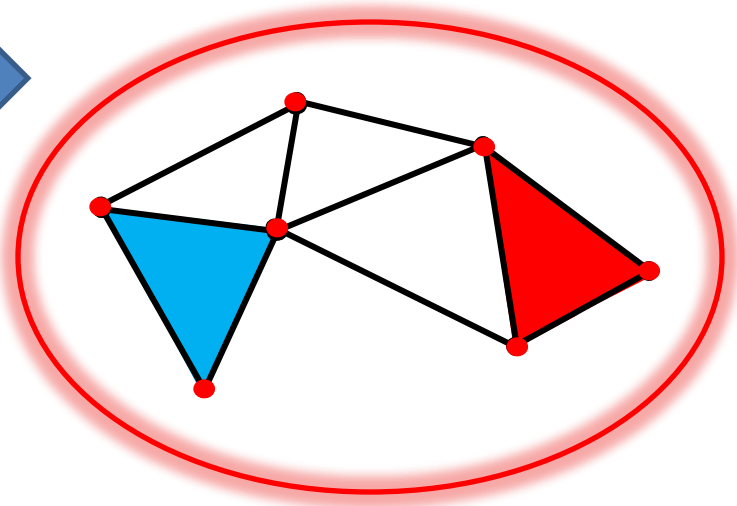
This is what we need

Čech complex – between space and time

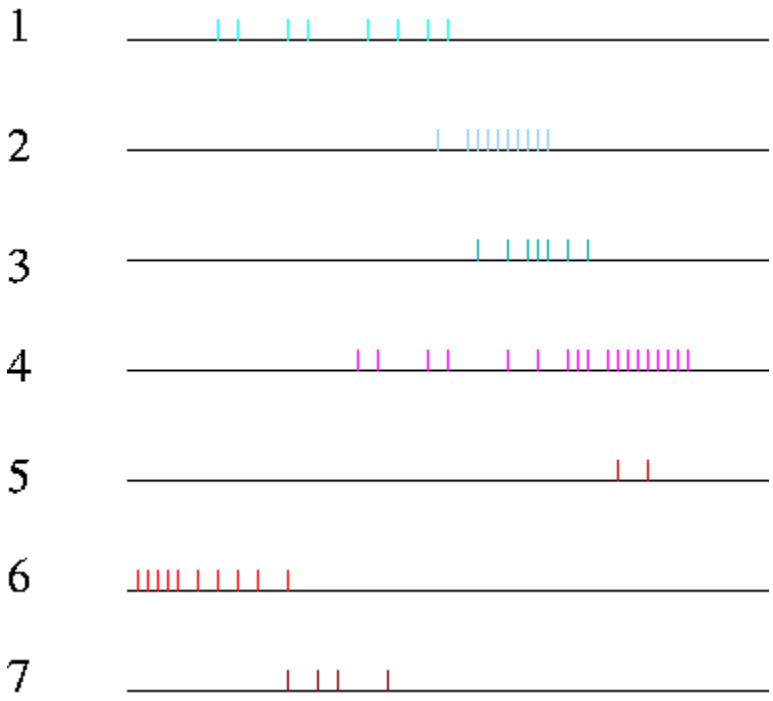


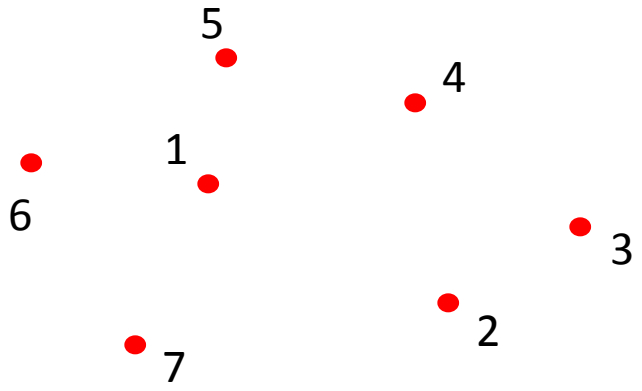
Place field map

Simplicial complex



Temporal Čech complex





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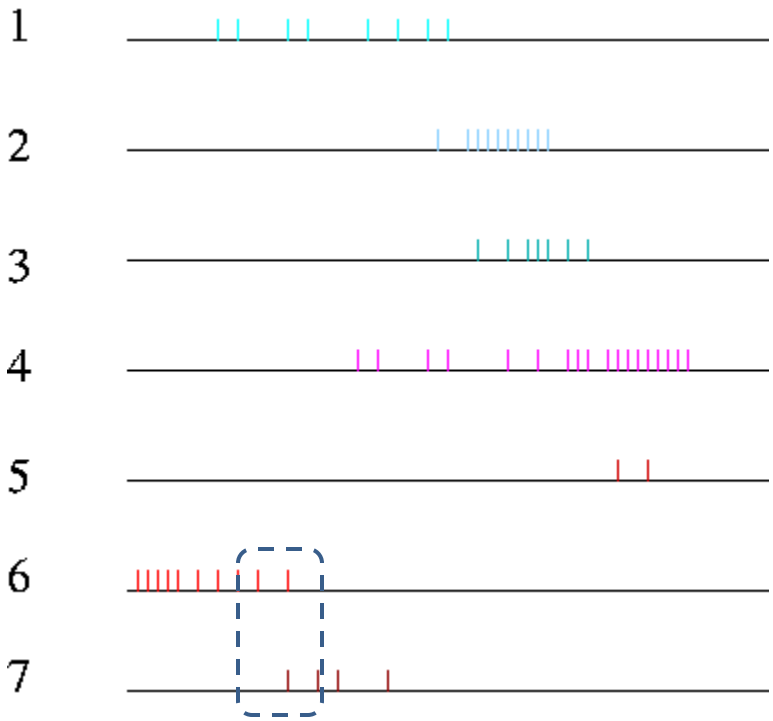

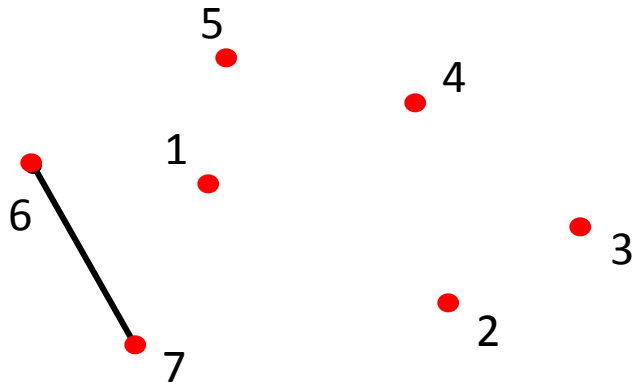

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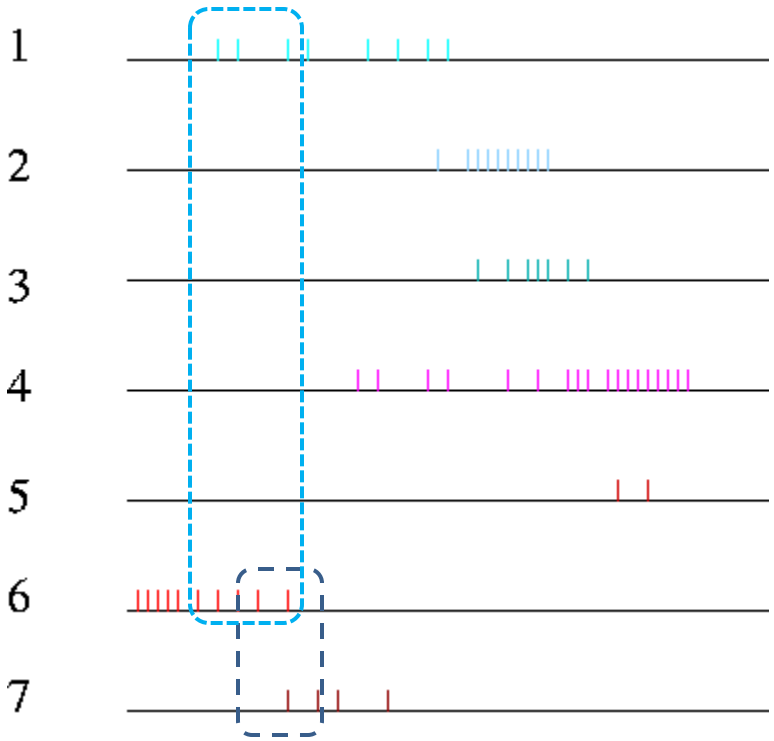

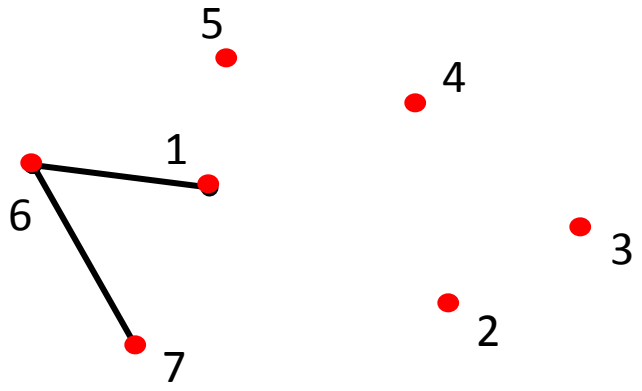

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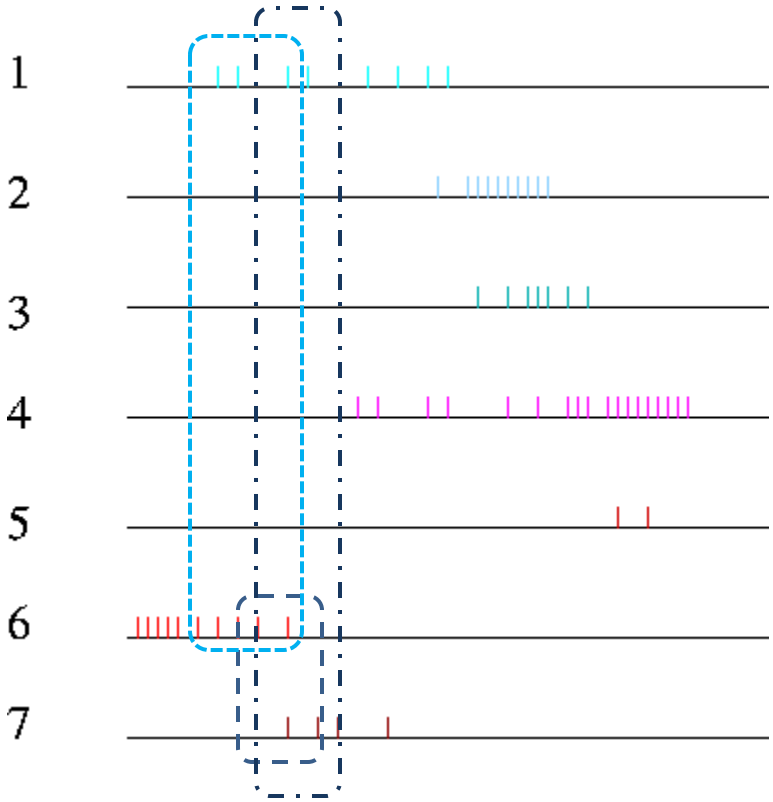

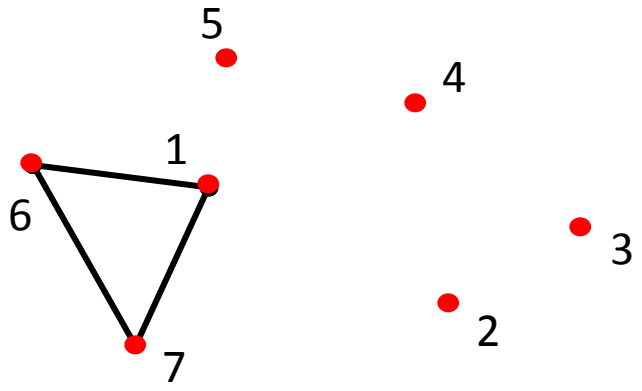

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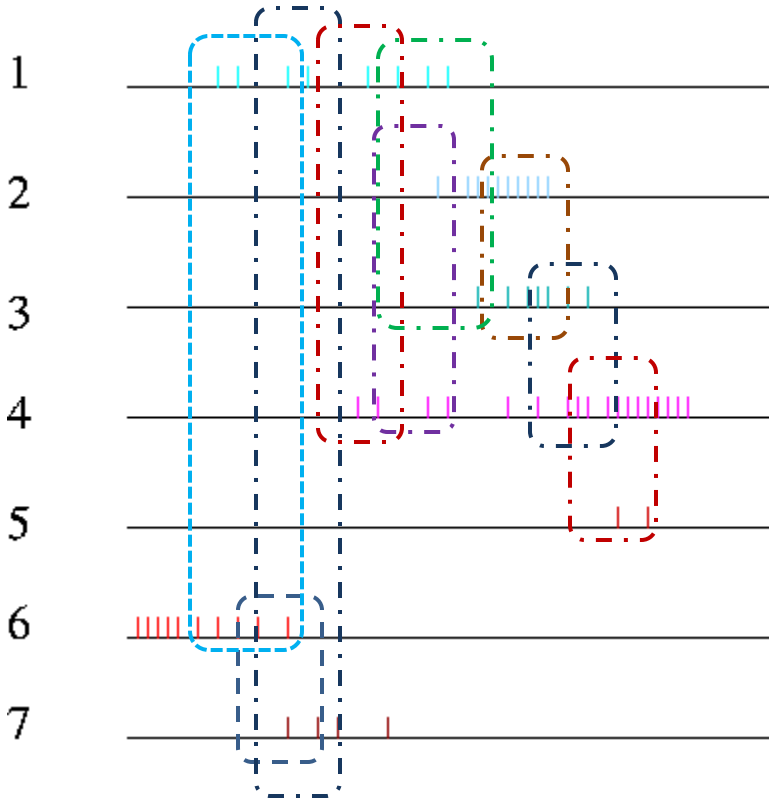

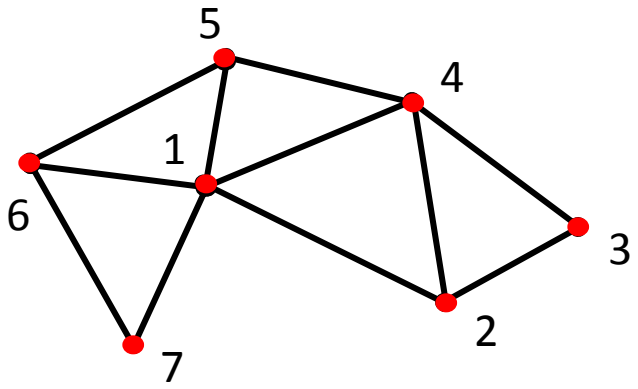

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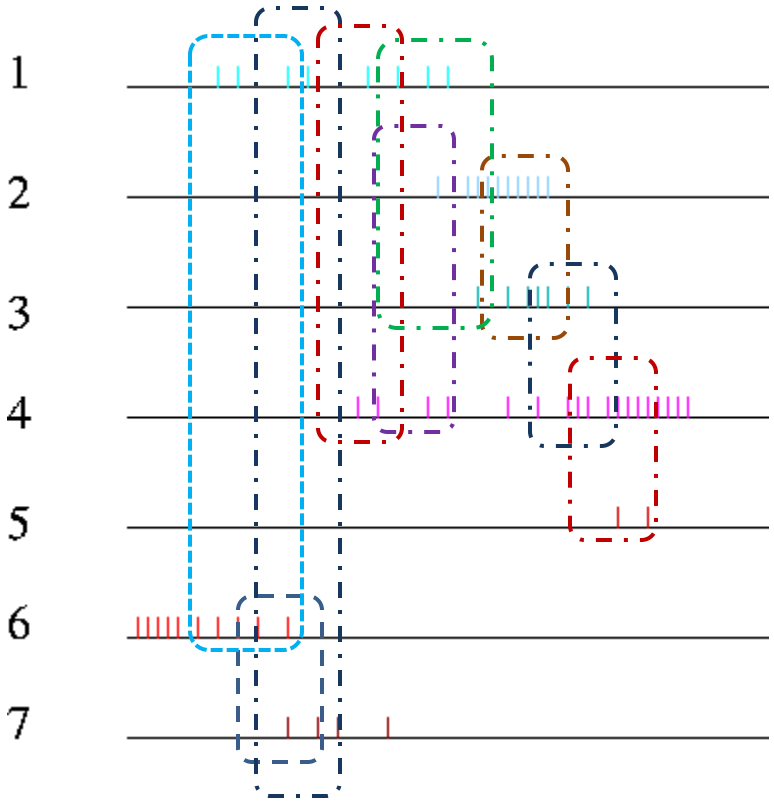
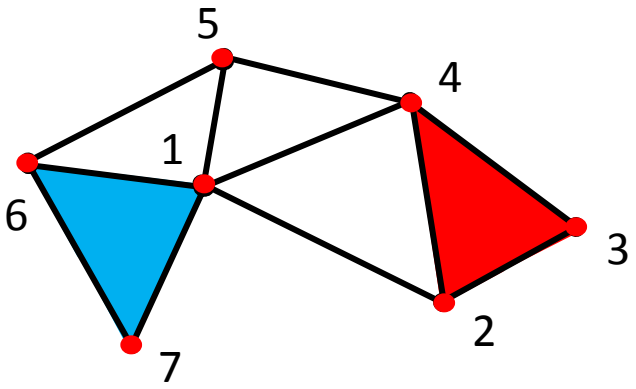
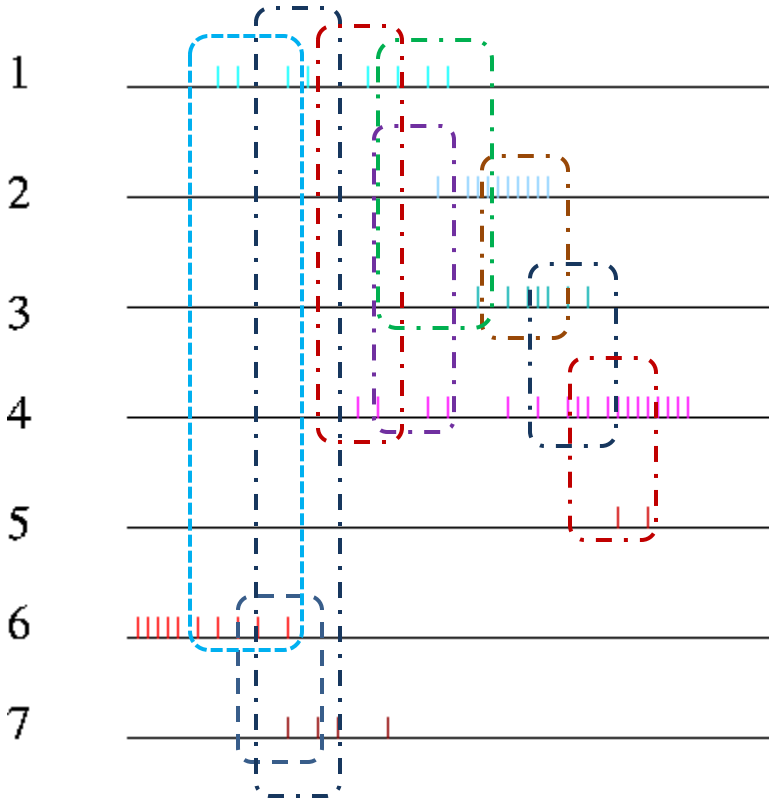
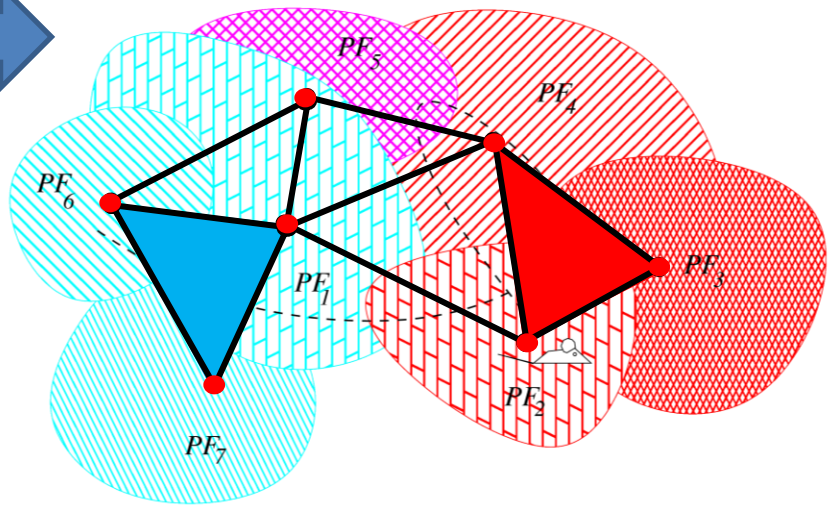




Čech complex – between space and time

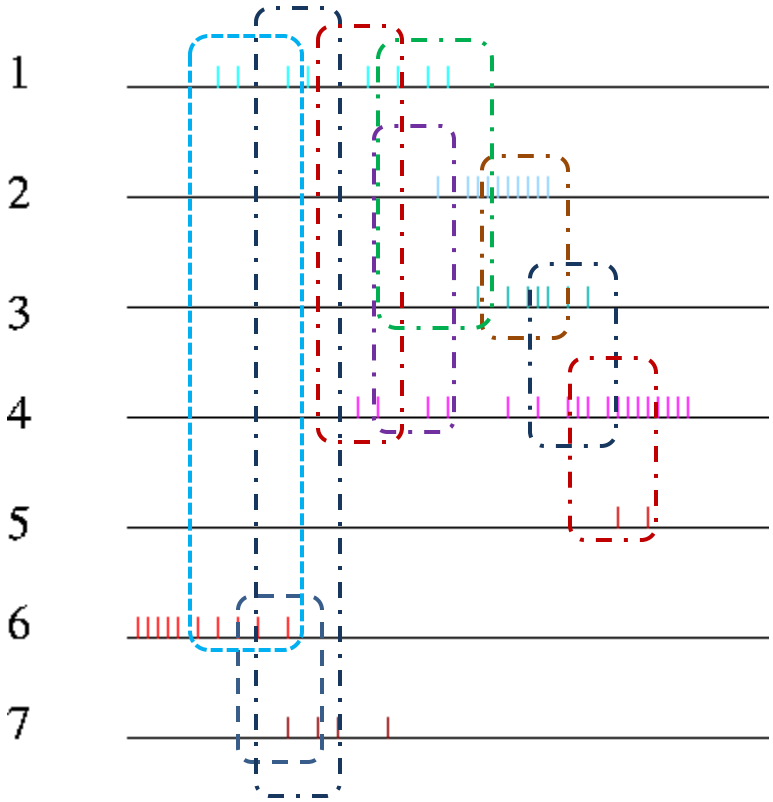


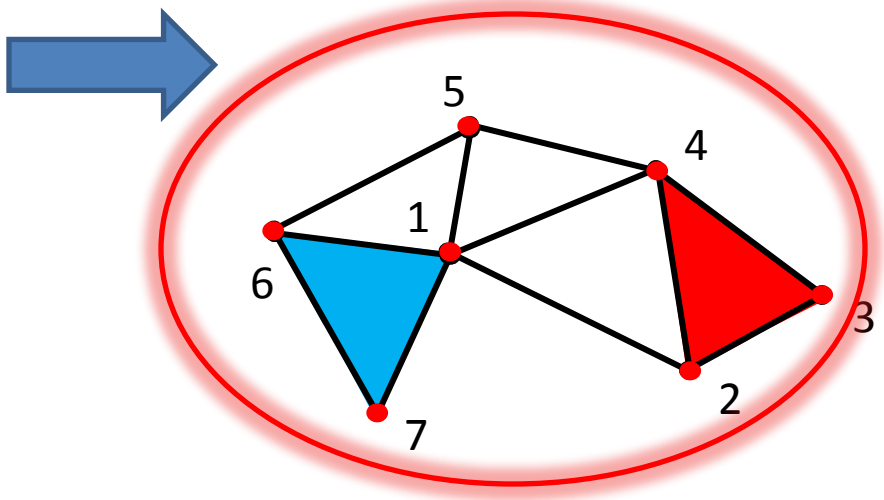


Temporal Spatial simplicial complex

Dabaghian et al., (2007)

Temporal Čech complex

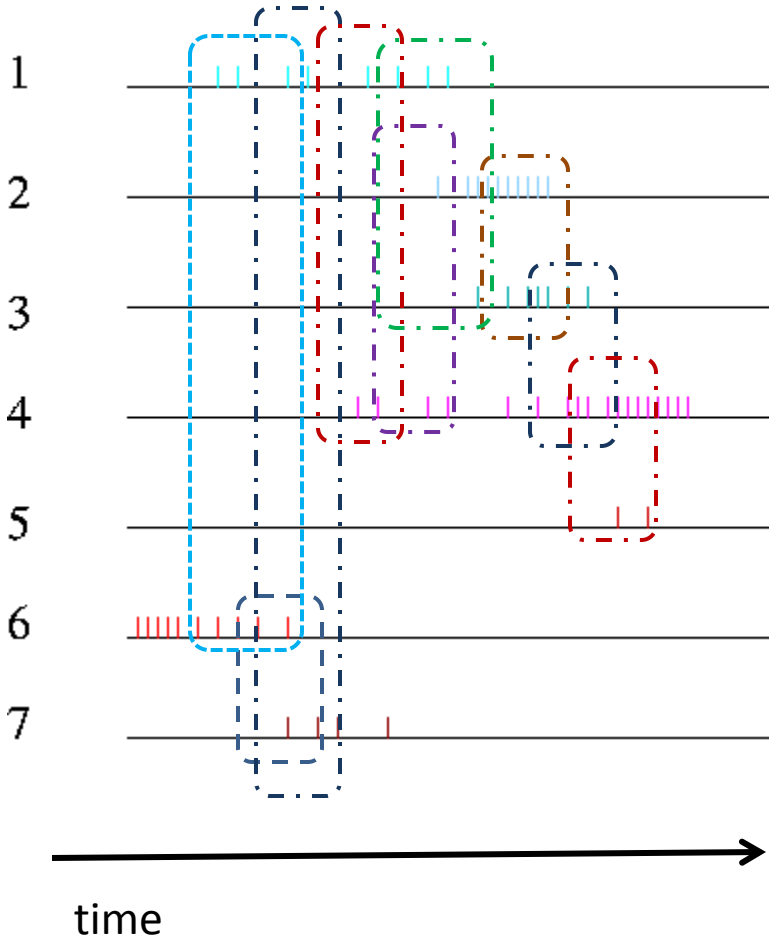




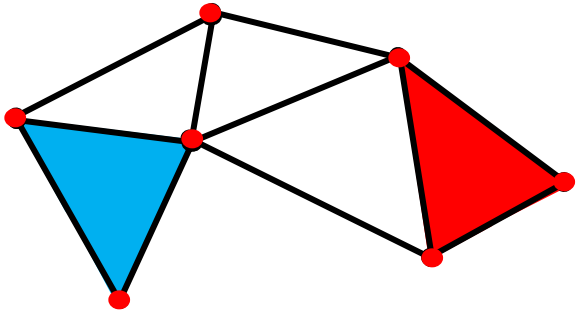
Simplicial complex

Dabaghian et al., (2007)

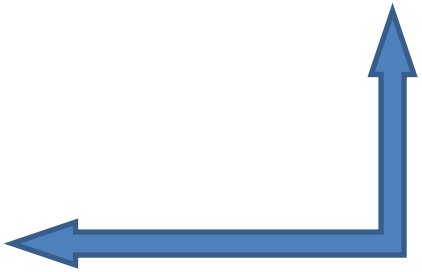
Topological information unfolds in time



Accumulation of spikes

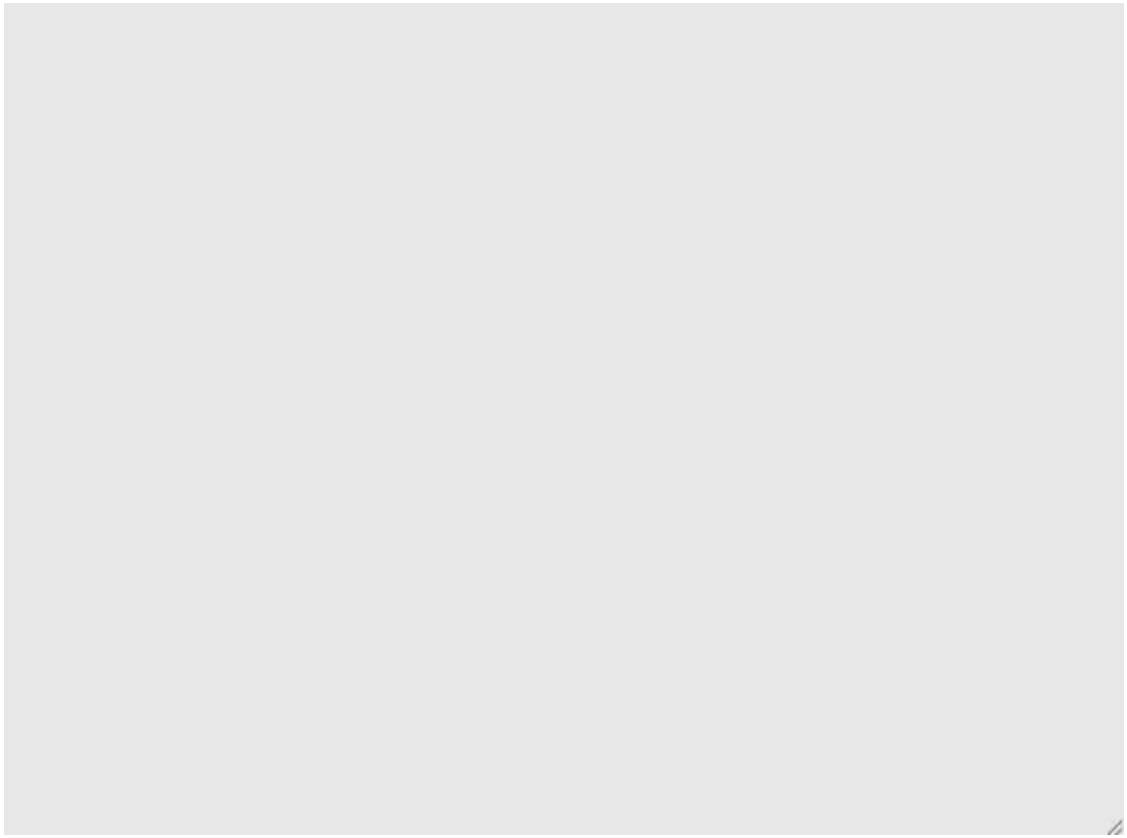


Accumulation of topological information



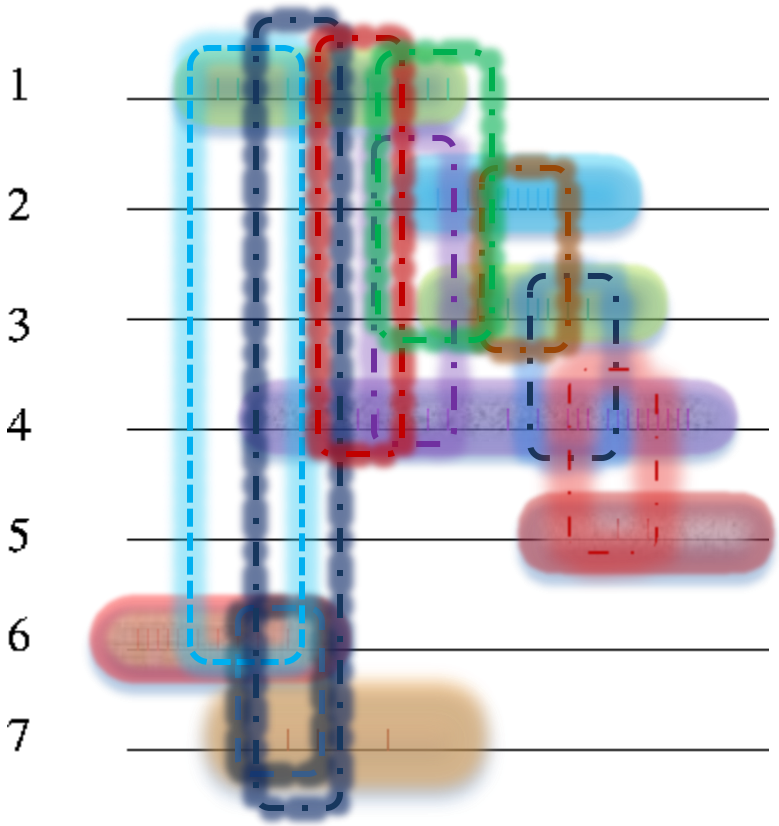
Dabaghian et al, (2008)

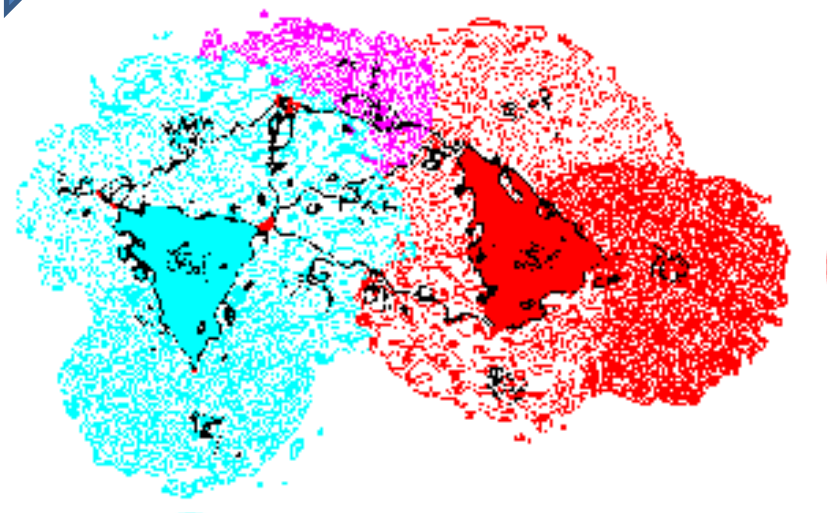
Temporal coactivity → place field connectedness



Can this actually work?

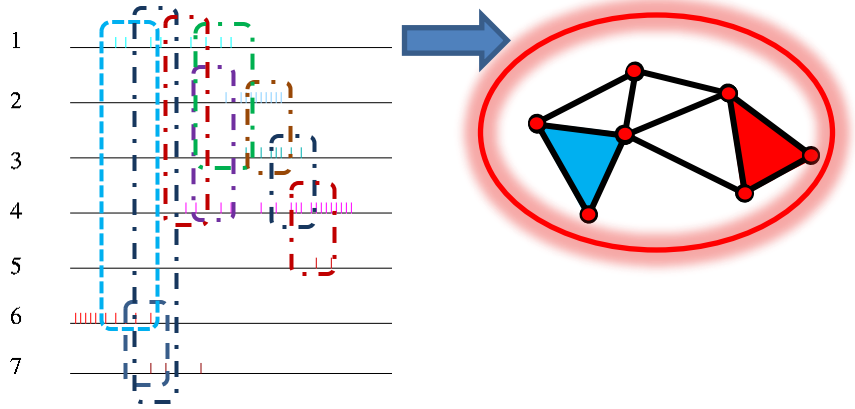
Can the map sustain biological variability?





Can this actually work?

Topics for topological analysis



- Is topology always extractible?
- Are the results robust?
- Does it take a reasonable time?

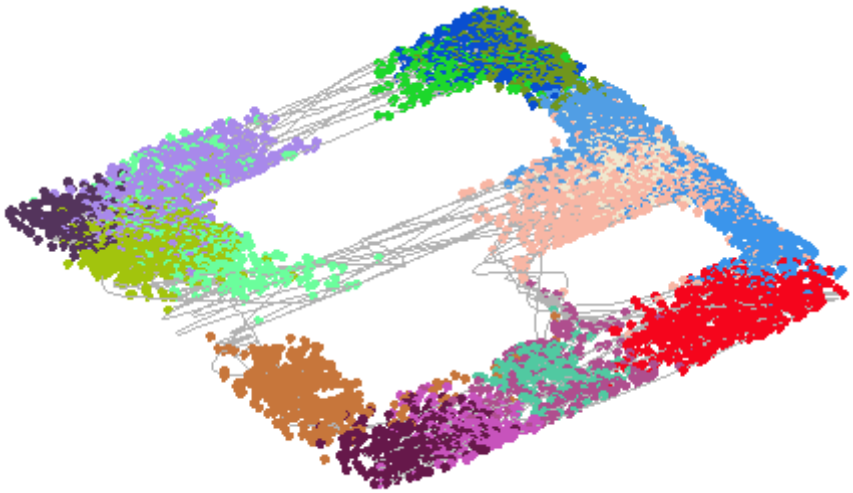
Answers depend on the quality of a map

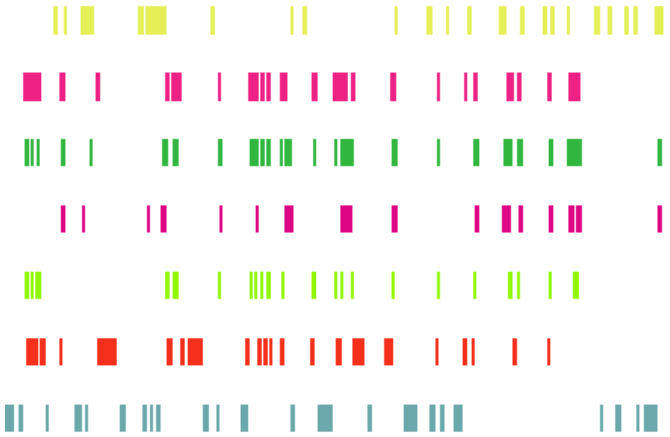
Quality of the spatial map

Map characteristics:

- **Firing rates of place cells**
- **Place field sizes**
- Etc.

$$\langle F \rangle, \langle s \rangle, N$$





Quality of the spatial map

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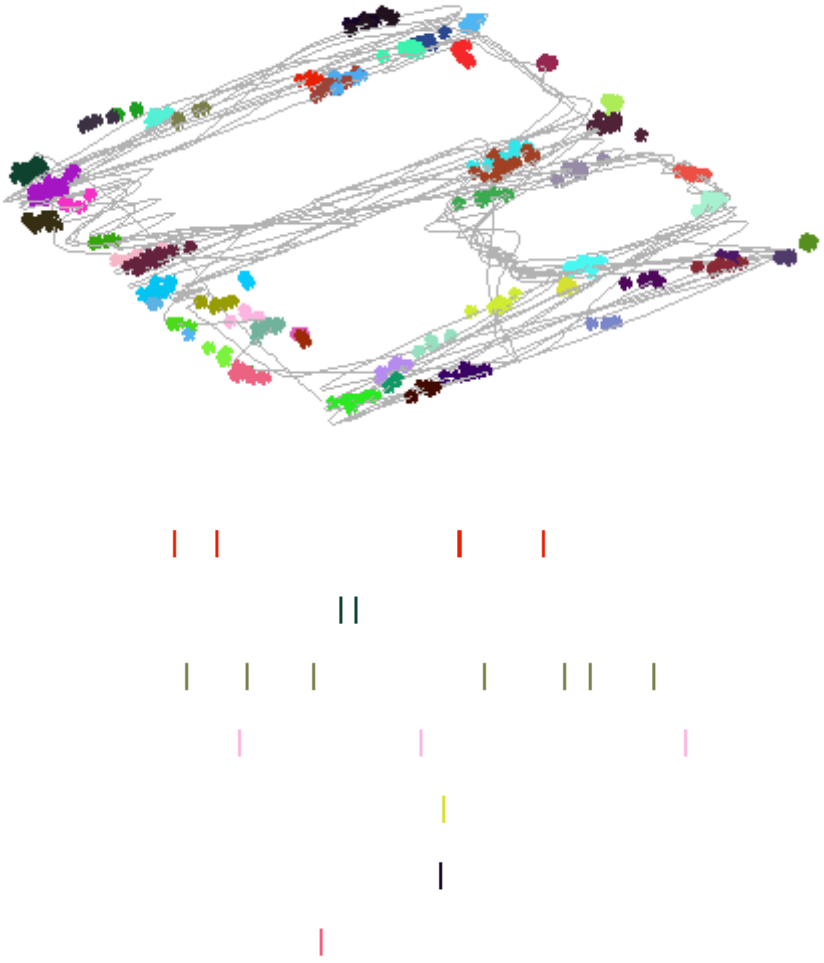
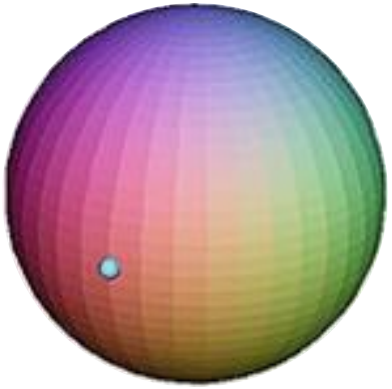
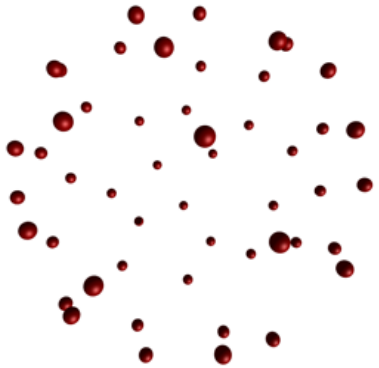
$$\langle F \rangle, \langle s \rangle, N$$



Topological persistence

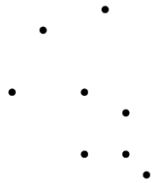


Topological persistence

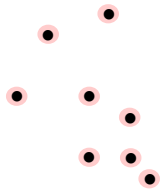


Topological persistence

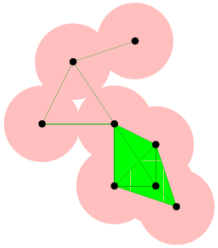
POINTS

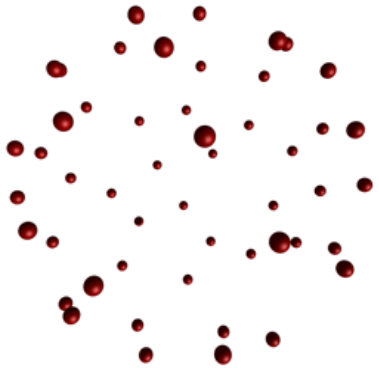


ε-Balls

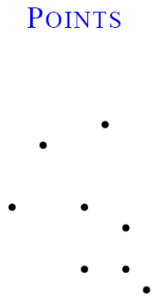


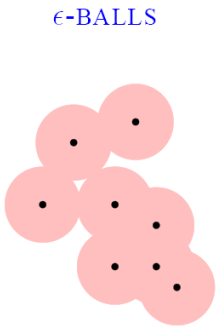
ČECH COMPLEX

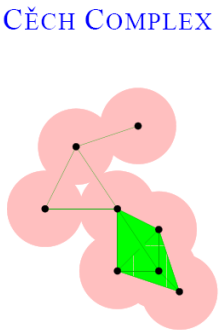


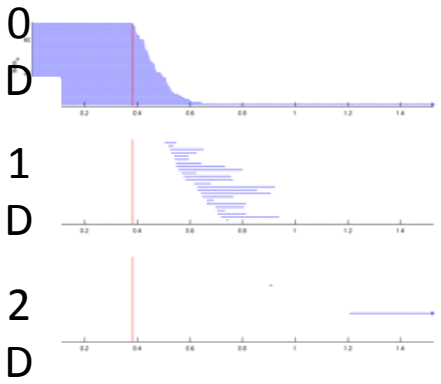


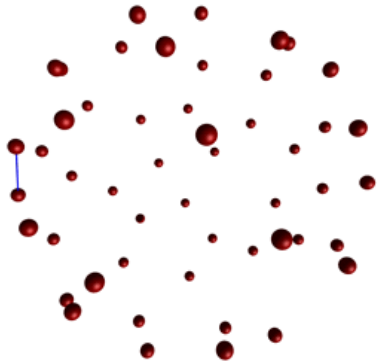
Topological persistence

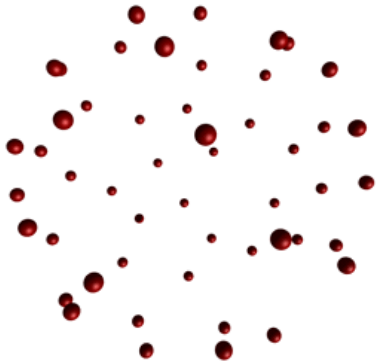




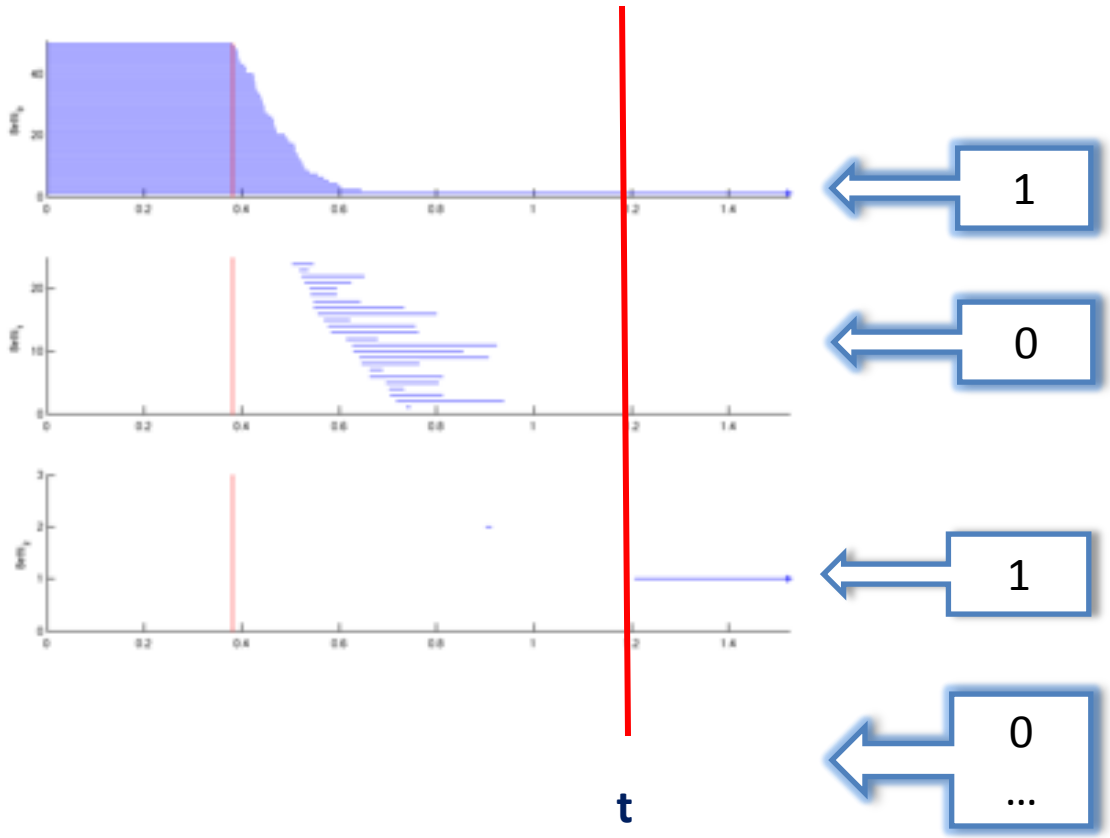




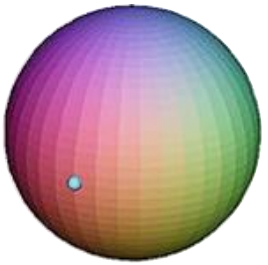




Topological barcode of a sphere



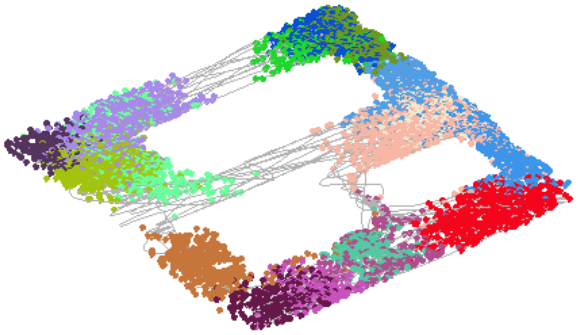
→ Sphere →

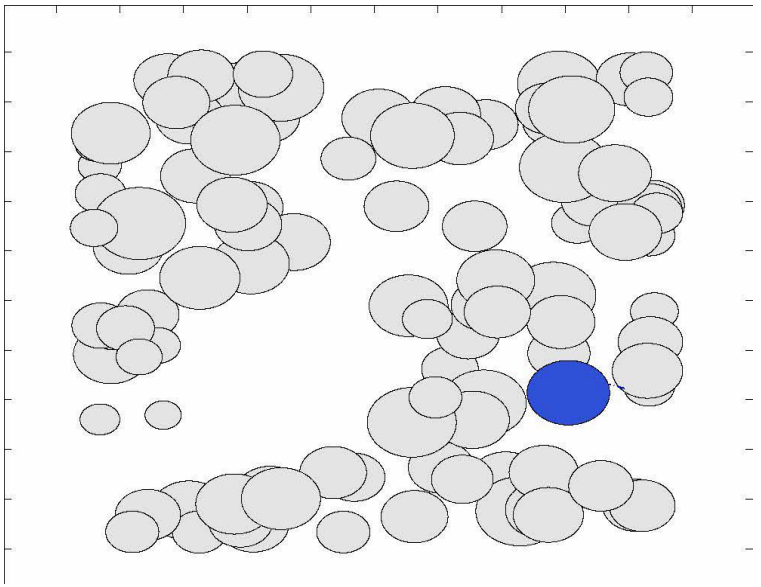


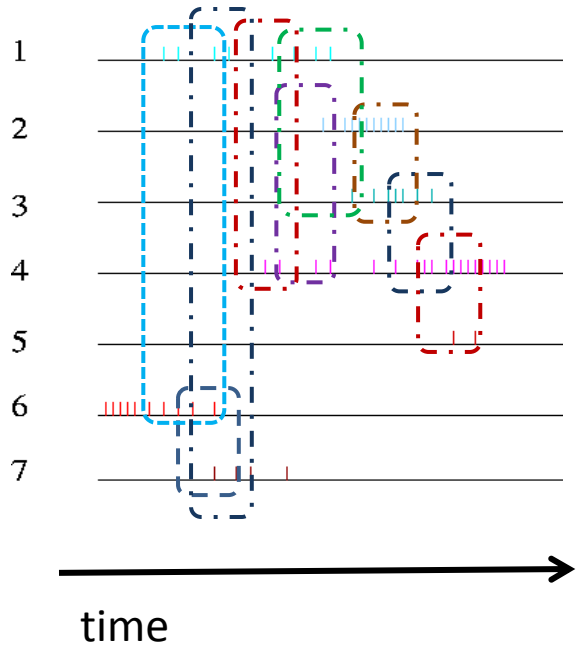
Topological barcode (1, 0 , 1, 0, 0 …)

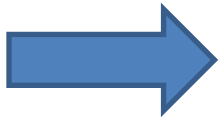
Computational test of a map's

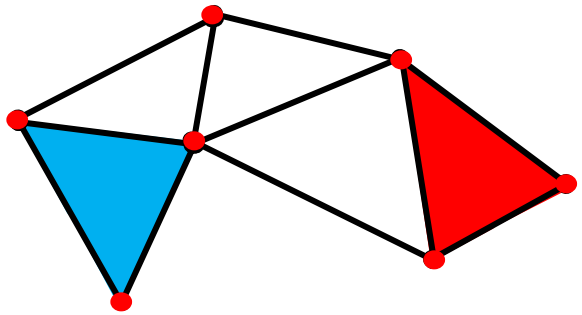
A map model (rate and size distributions)











Computational test of a map's

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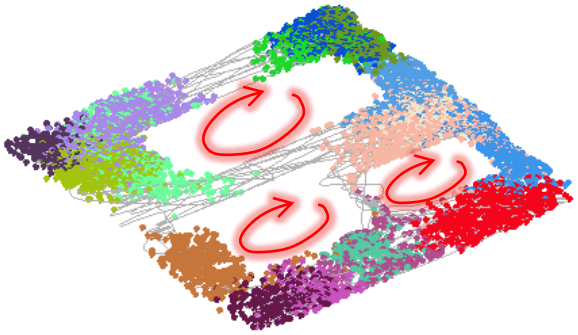
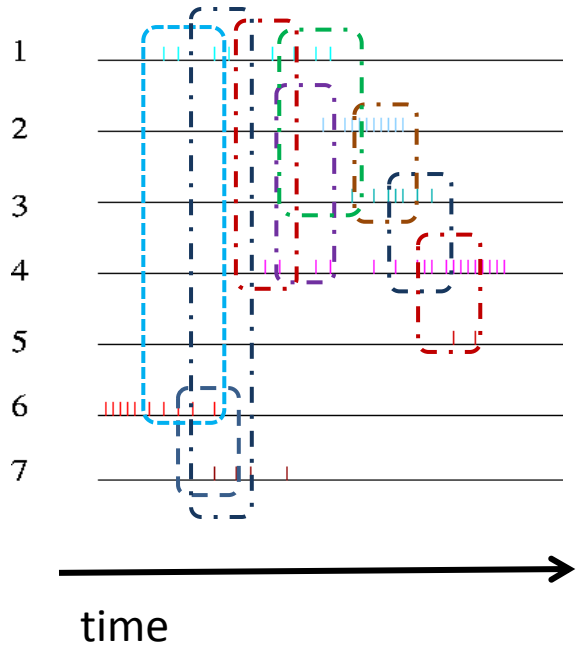
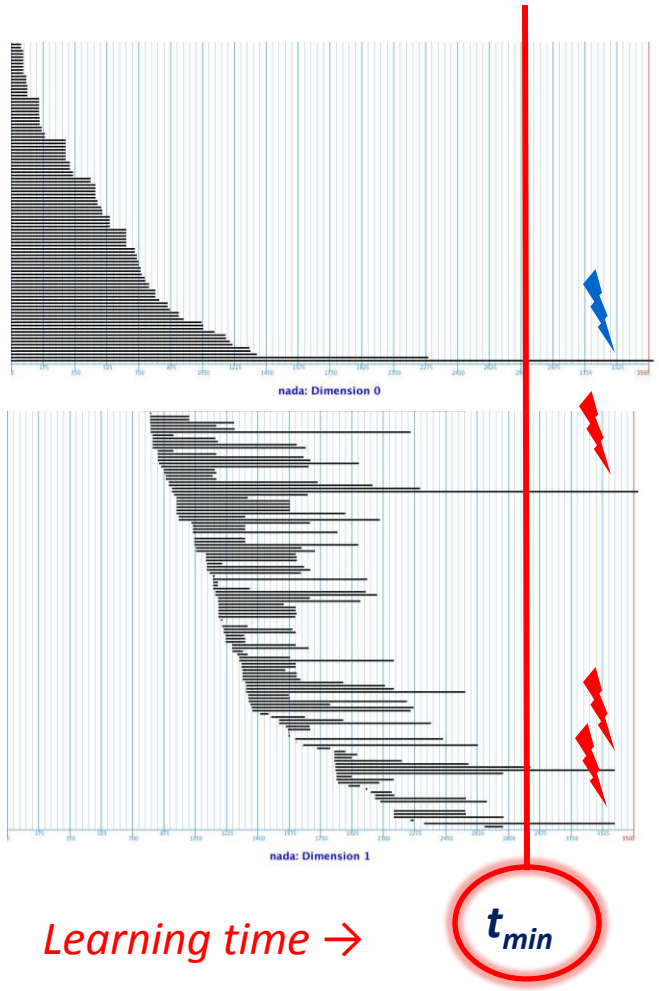




Persistent homology, loop dynamics



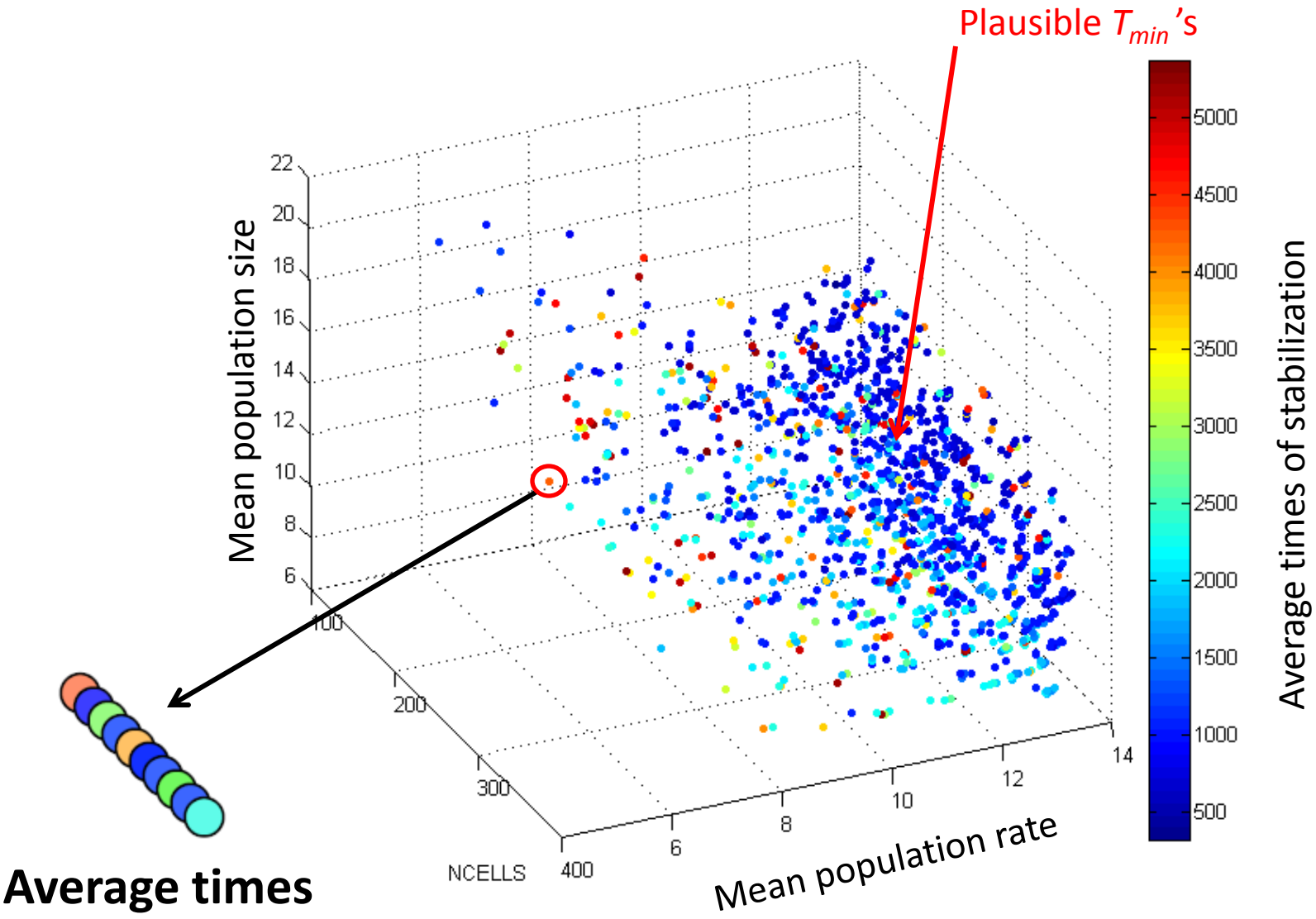
Learning time via map parameters

$$T_{min} = T_{min}\left(\langle F \rangle, \langle s \rangle, N\right)$$

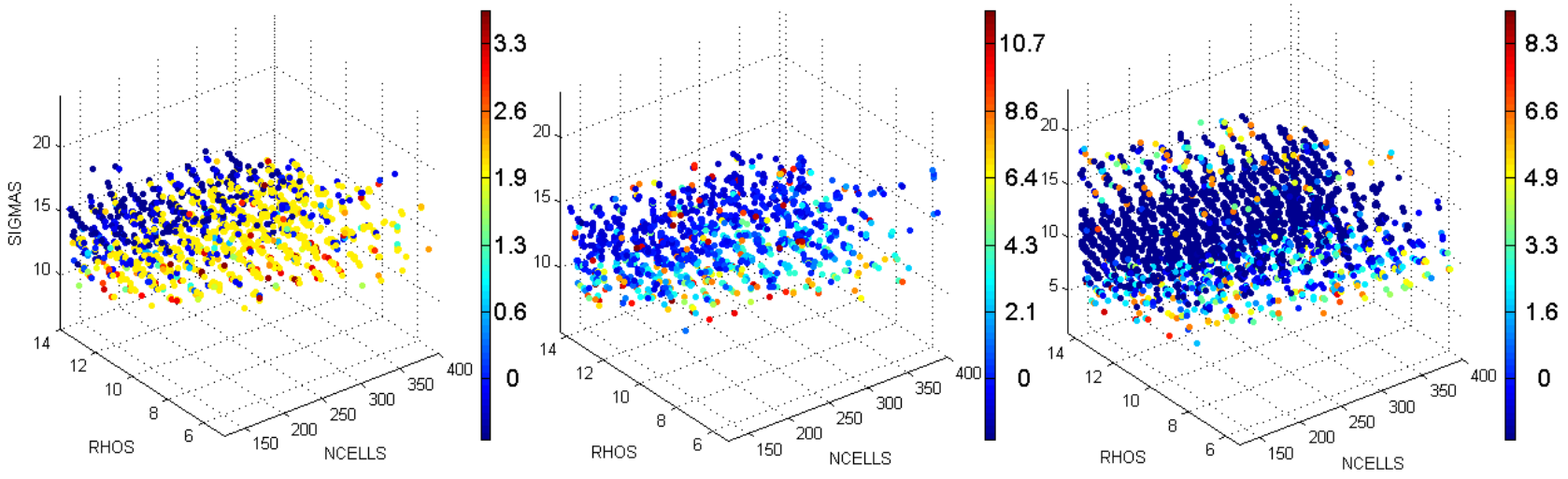
Learning time depends on map parameters

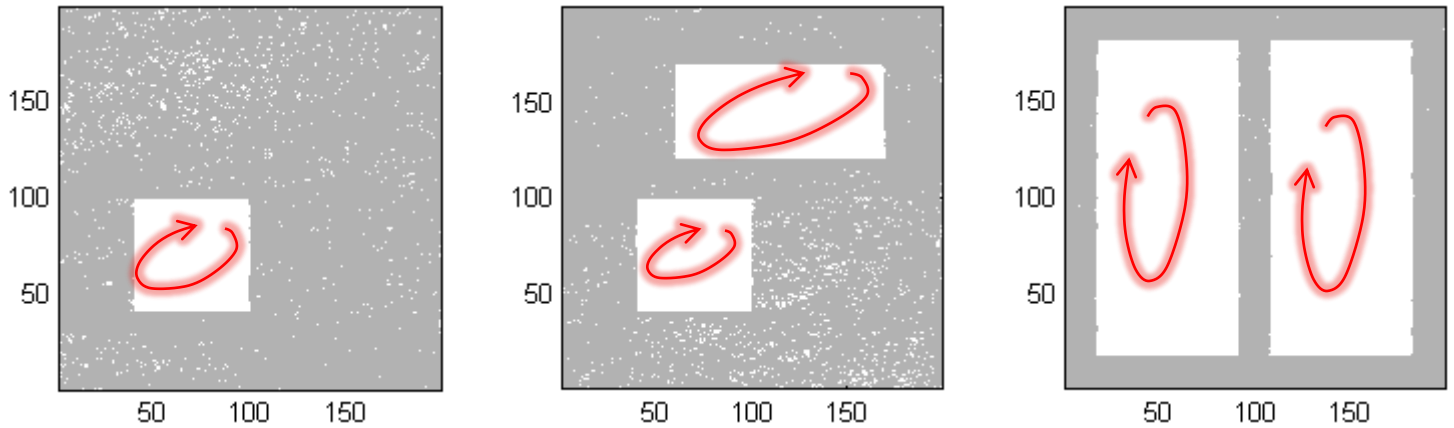
a. firing rate, $\langle F \rangle$
b. place field size, $\langle s \rangle$
c. number of cells, N

Minimal learning times $T_{min} = T_{min}\left(\langle F \rangle, \langle s \rangle, N\right)$



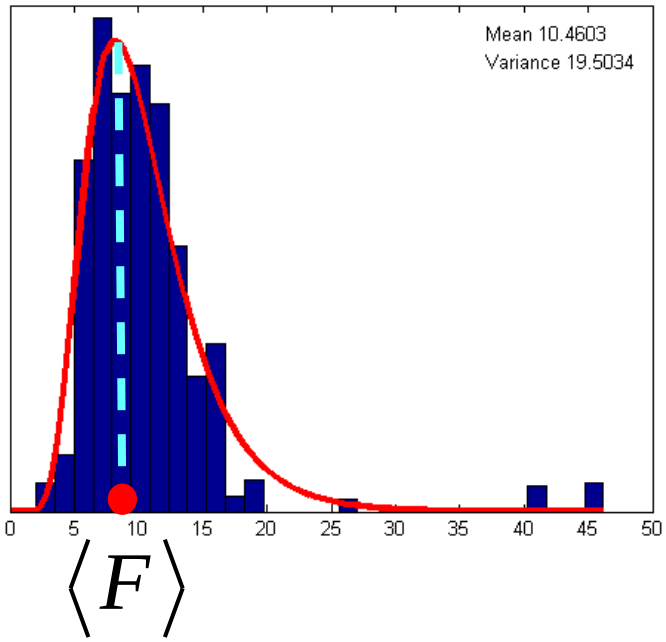
Minimal learning times T_{min}, different arenas





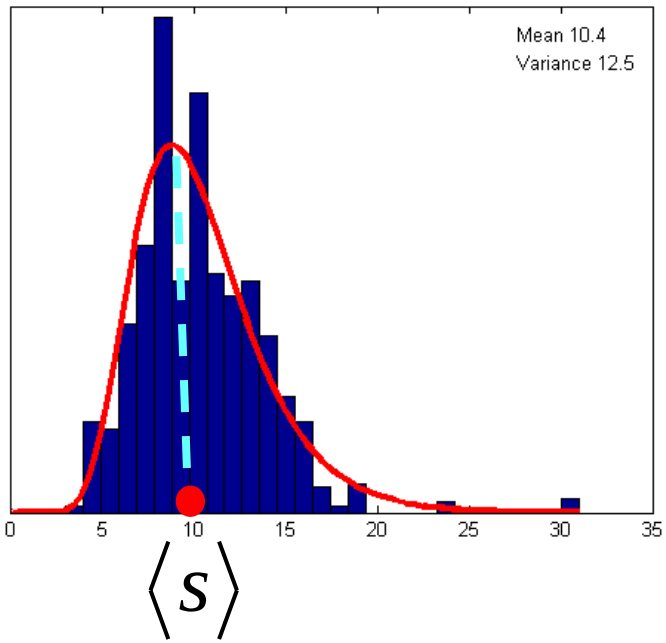
Place cell ensemble activity parameters

firing rates



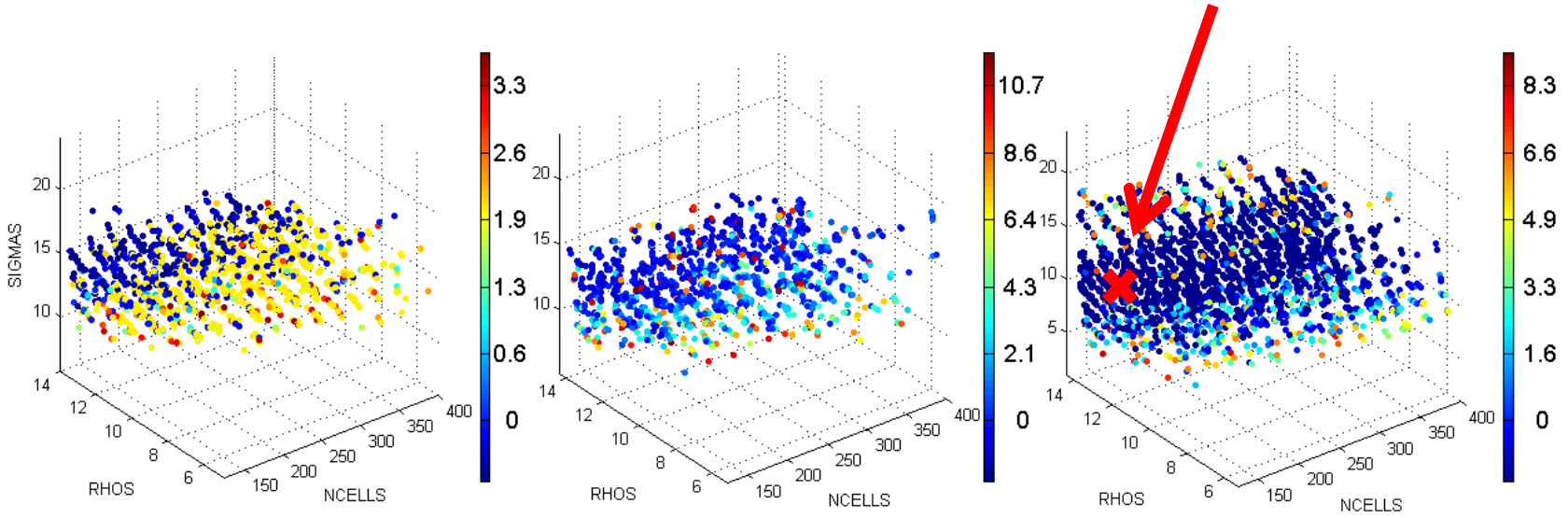
$\langle F \rangle$

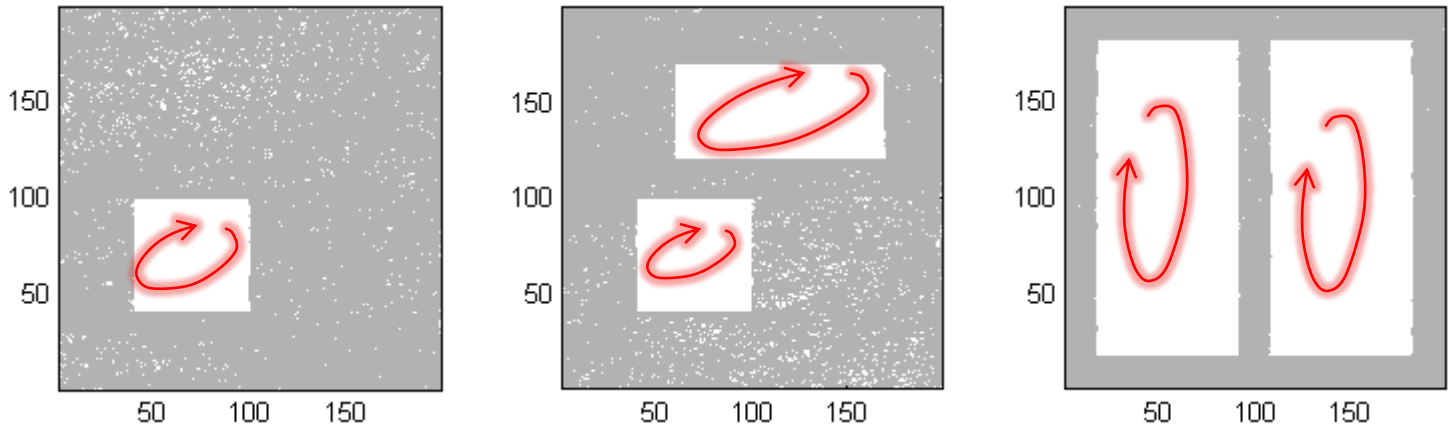
place field sizes



$\langle s \rangle$

Minimal learning times, T_{min}

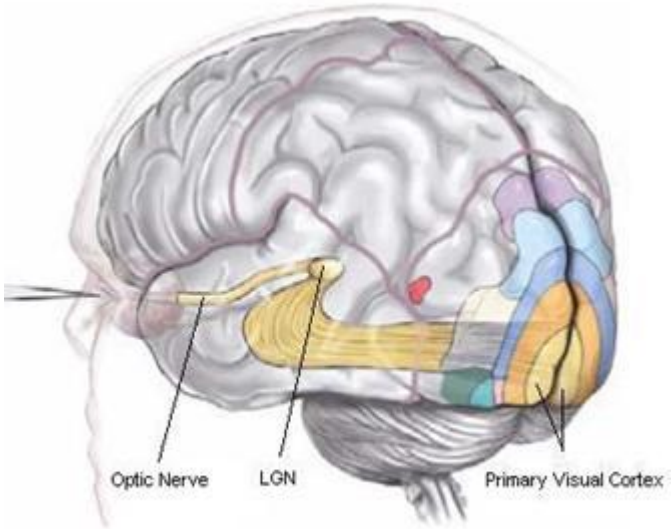


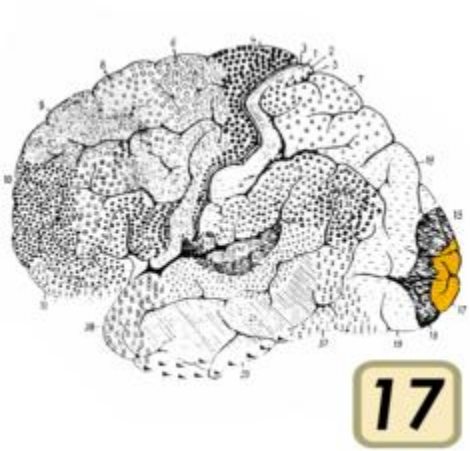


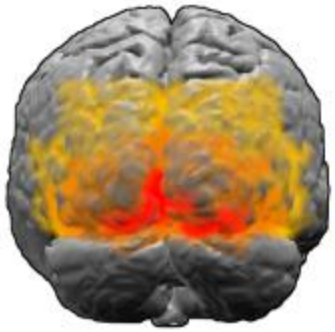
Predictive power of neuronal topology

1. Which parameters affect the map?
2. How does t_{min} change with the biological parameters?
3. How does t_{min} change with the geometry of the environment?
4. What is the “stability range” of the firing activity?
5. etc …

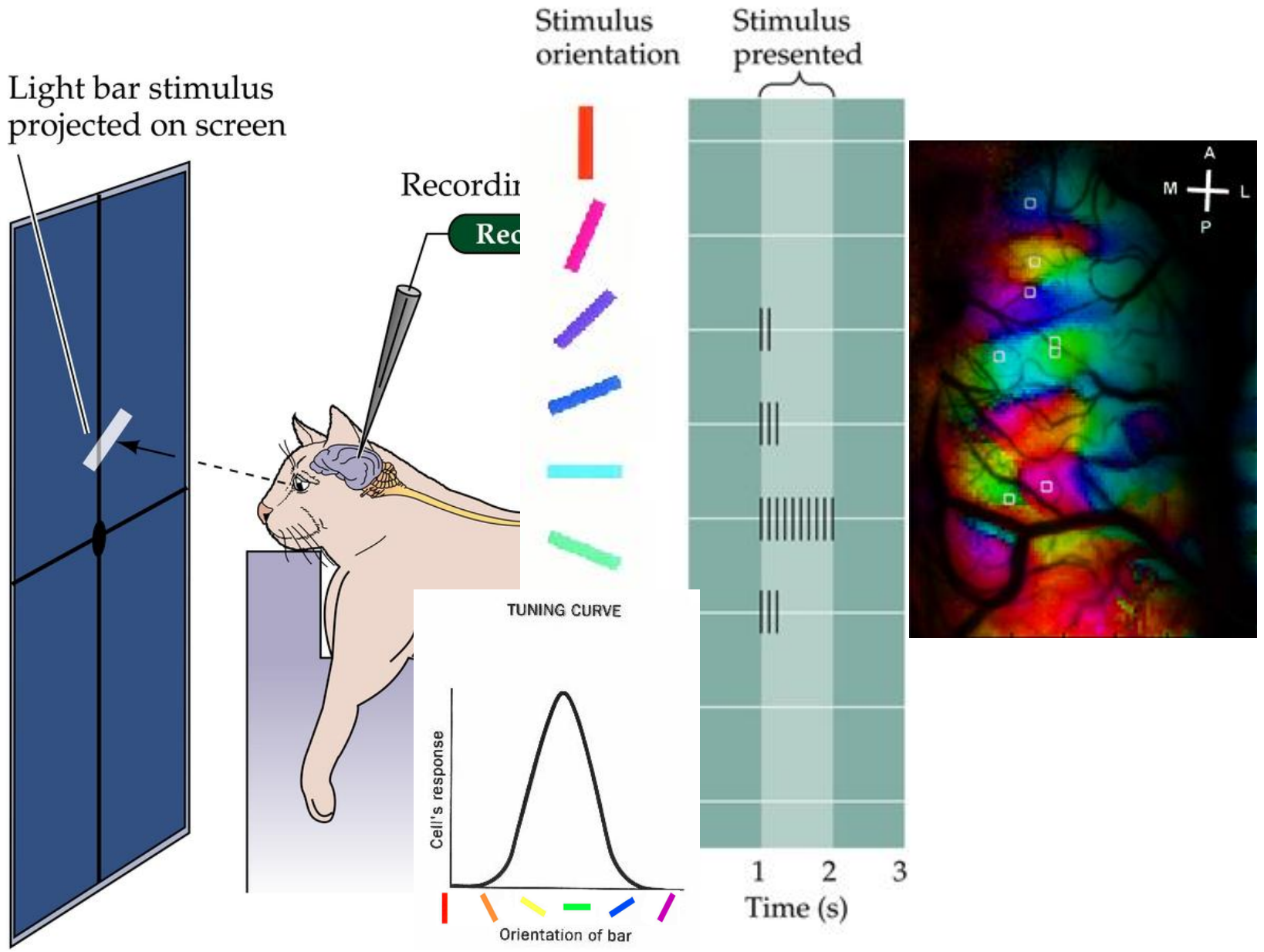
Visual cortex

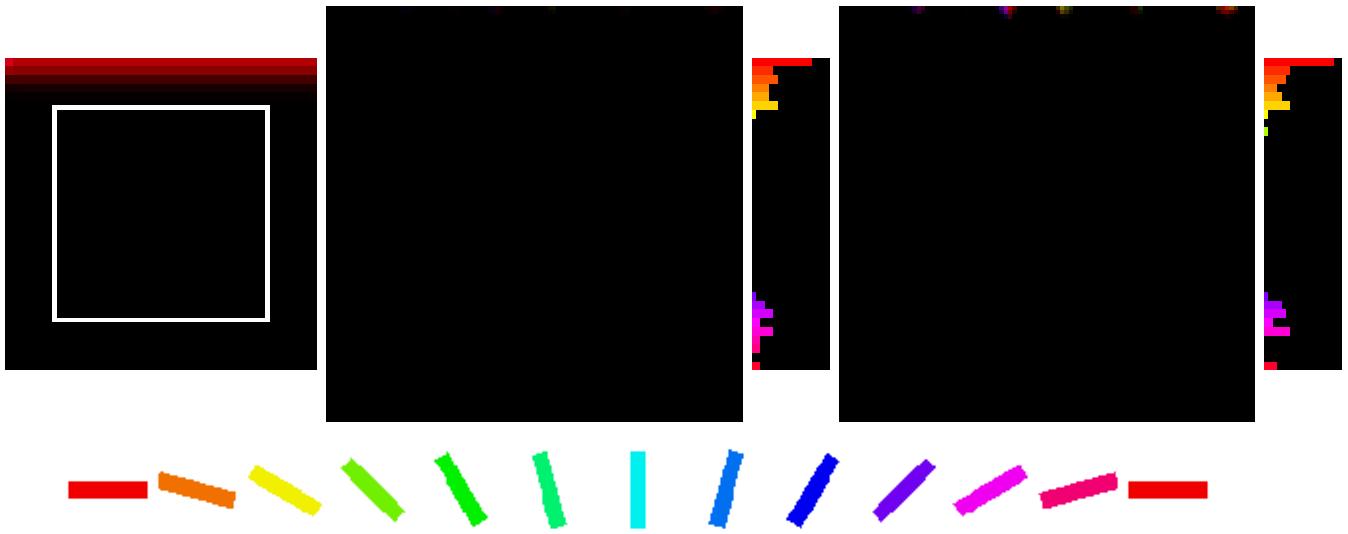




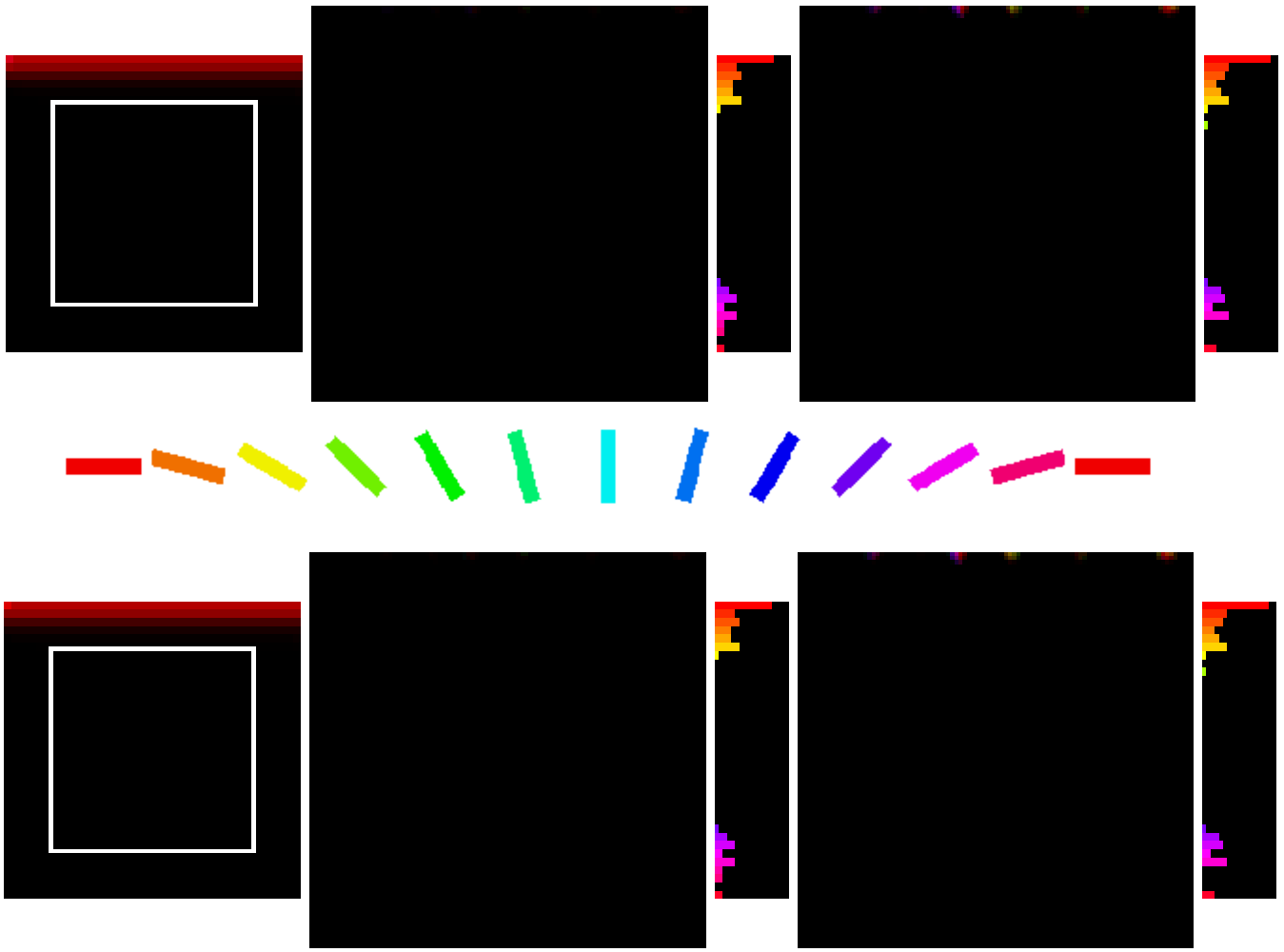


Visual cortex – orientation selectivity





http://homepages.inf.ed.ac.uk/jbednar/index.html



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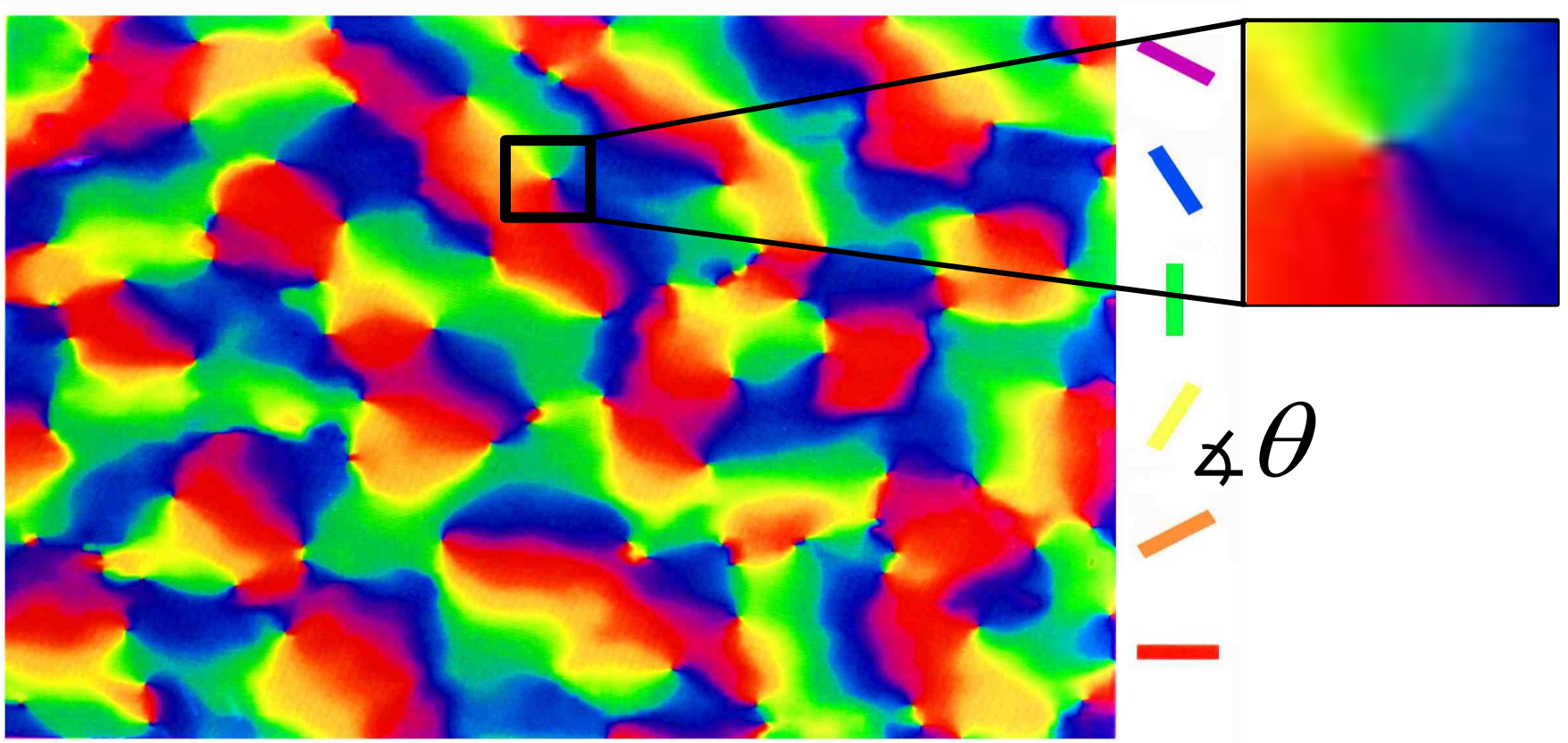
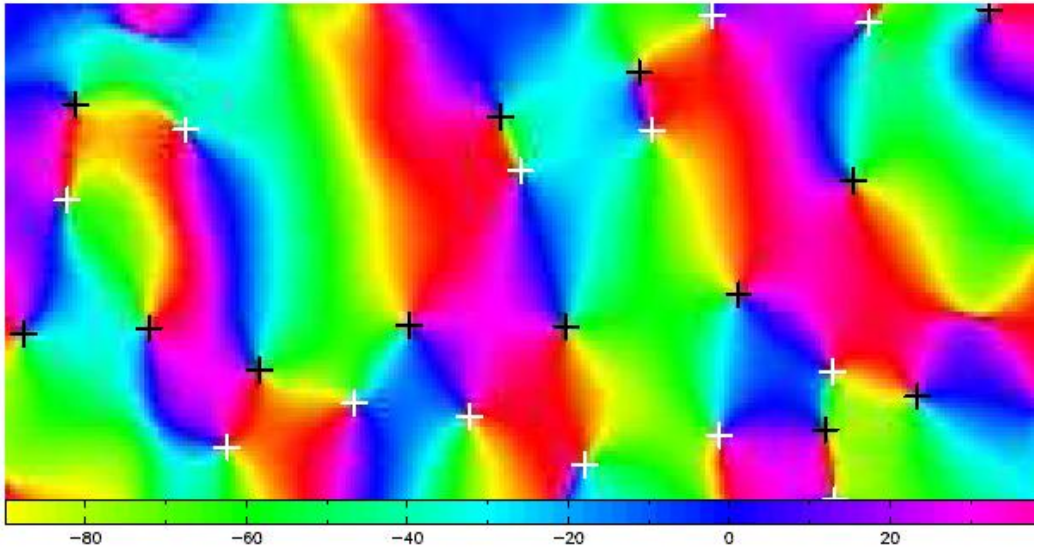
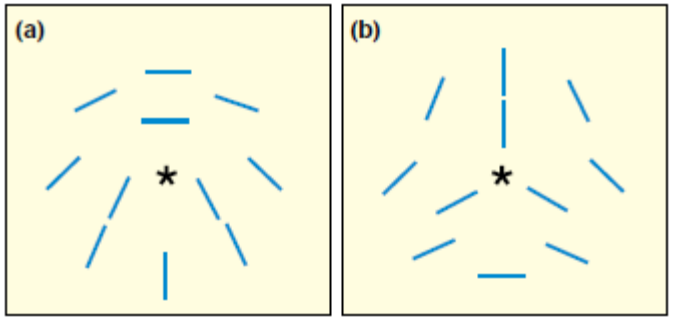
Visual cortex – orientation selectivity



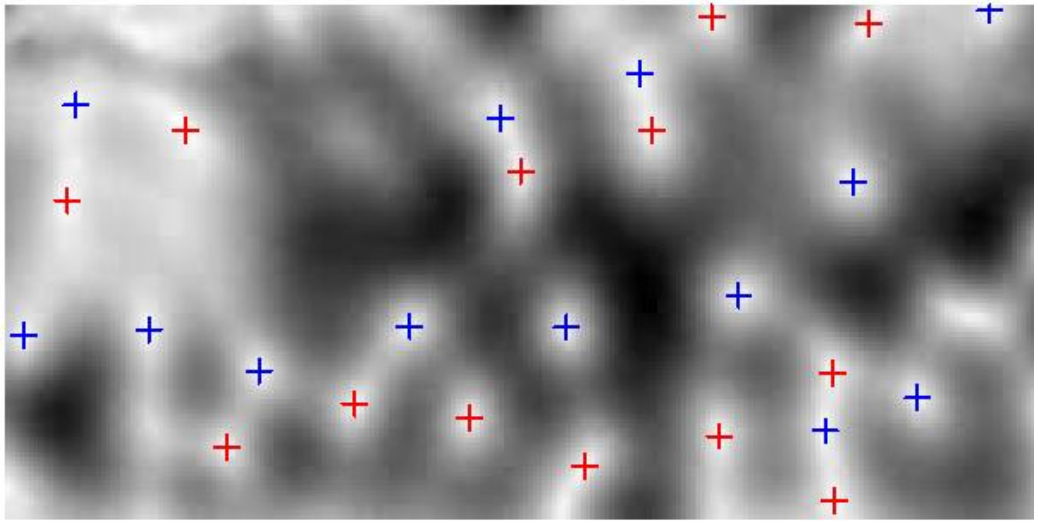
$$\theta = f(x, y)$$

Director field with topological singularities



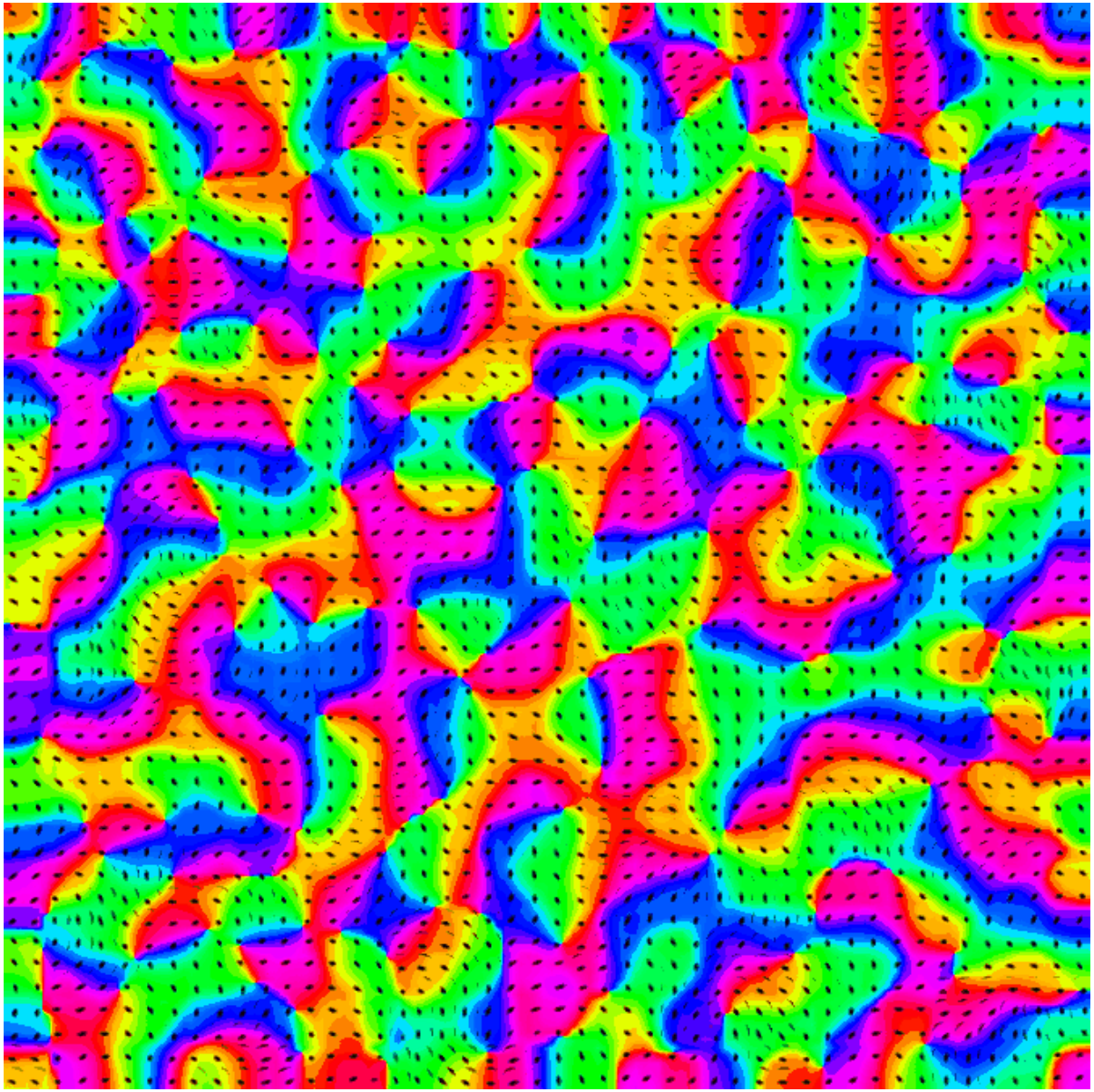


N. Swindale, 1996



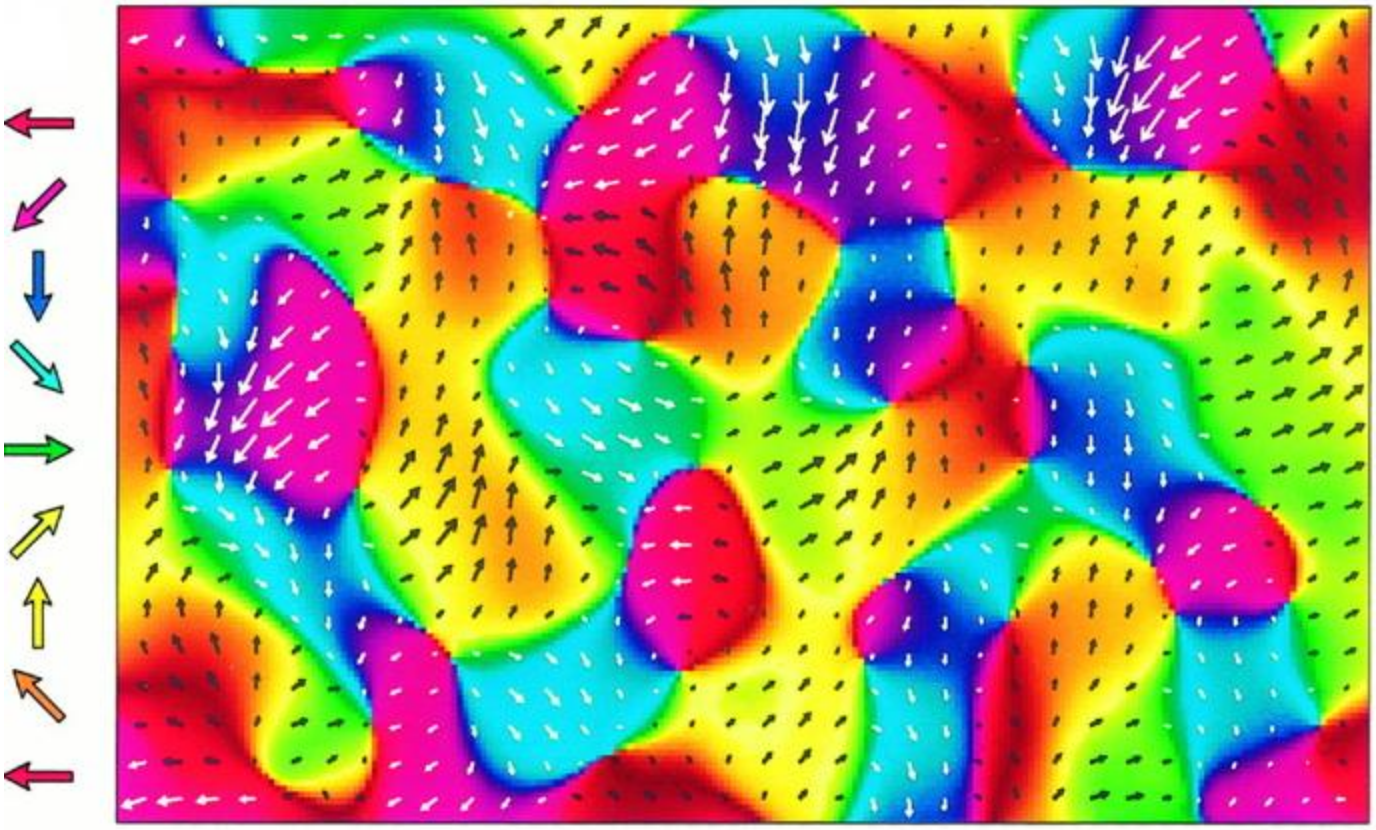
Topological charges of the director field singularities

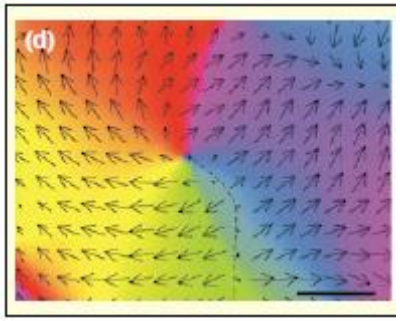
Courtesy of V. Kalatsky, UH

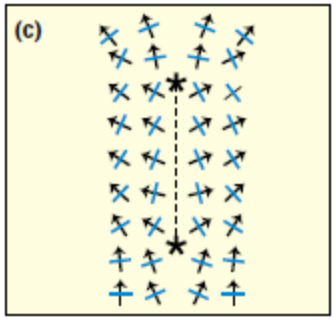


http://homepages.inf.ed.ac.uk/jbednar/index.html

Vector field with topological singularities

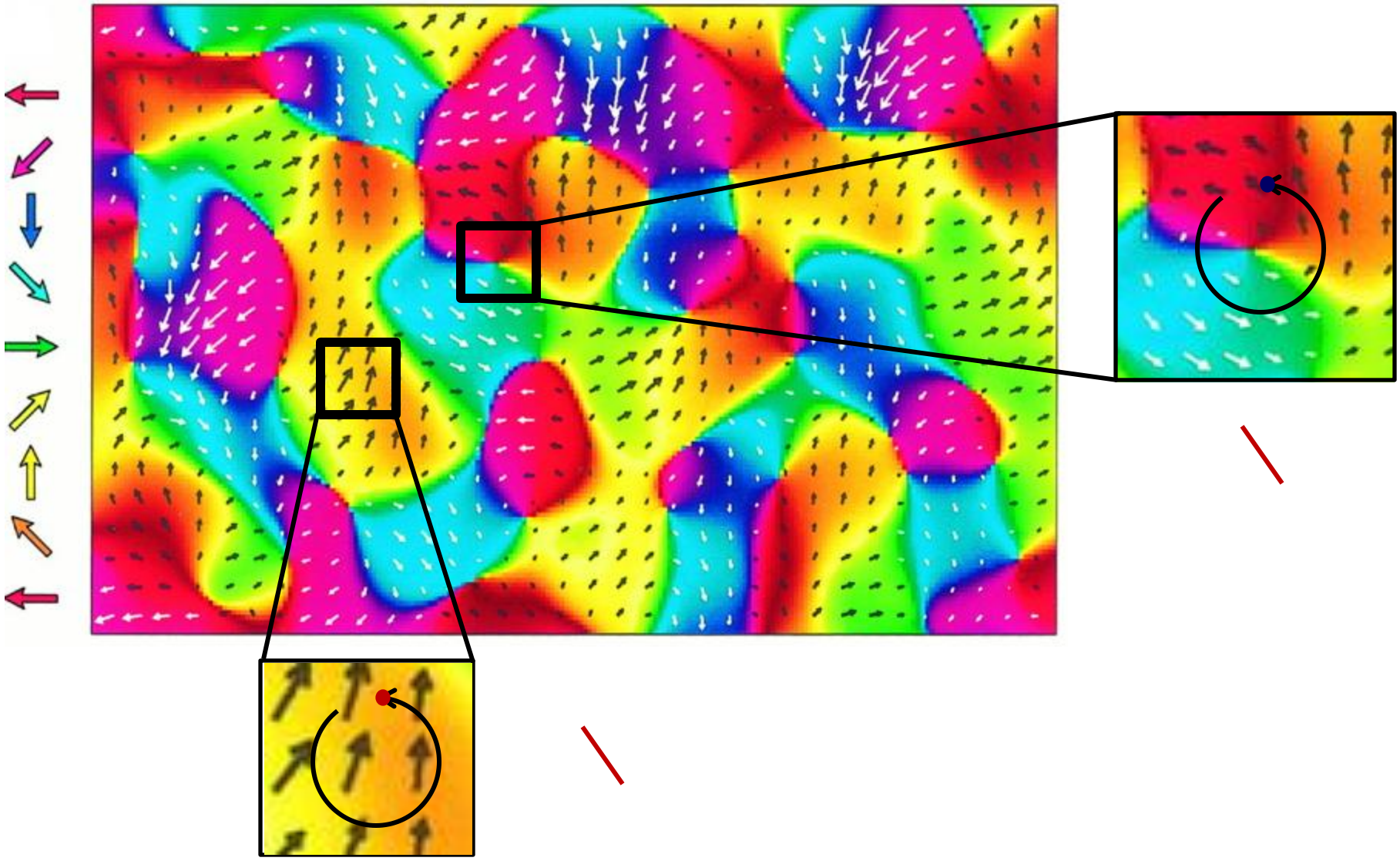




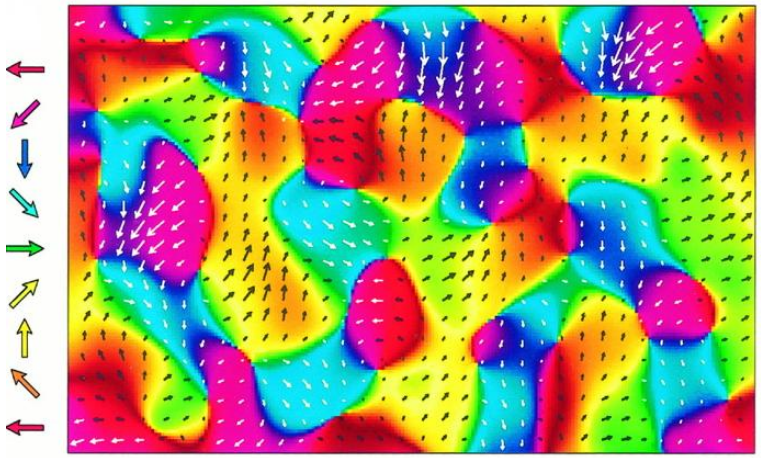


N. Swindale, 1996

Vector field with topological singularities



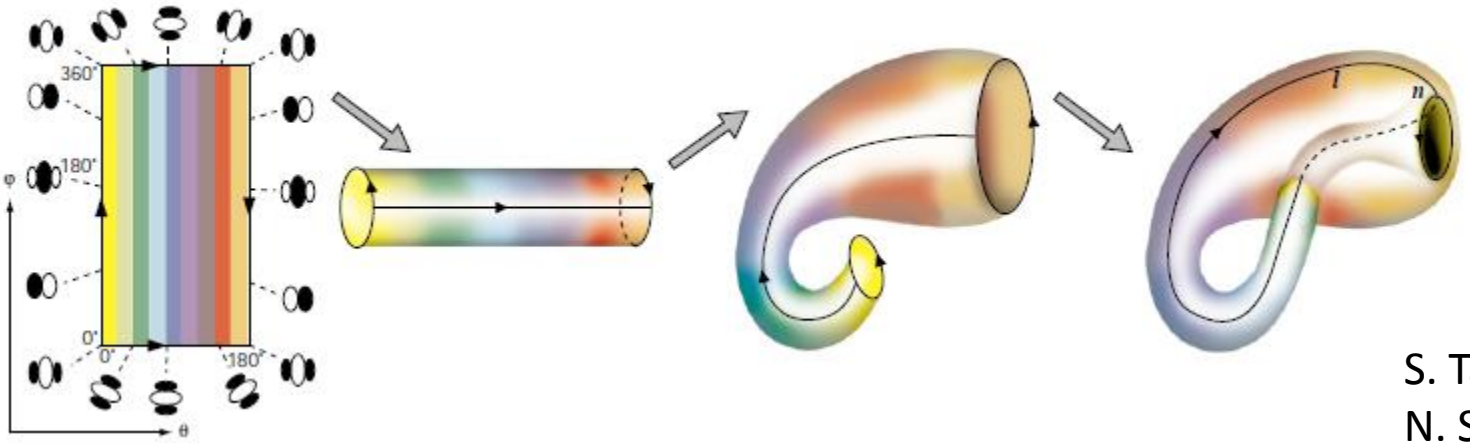
Dynamics of the vector field



Why so much geometry?

How is it used???

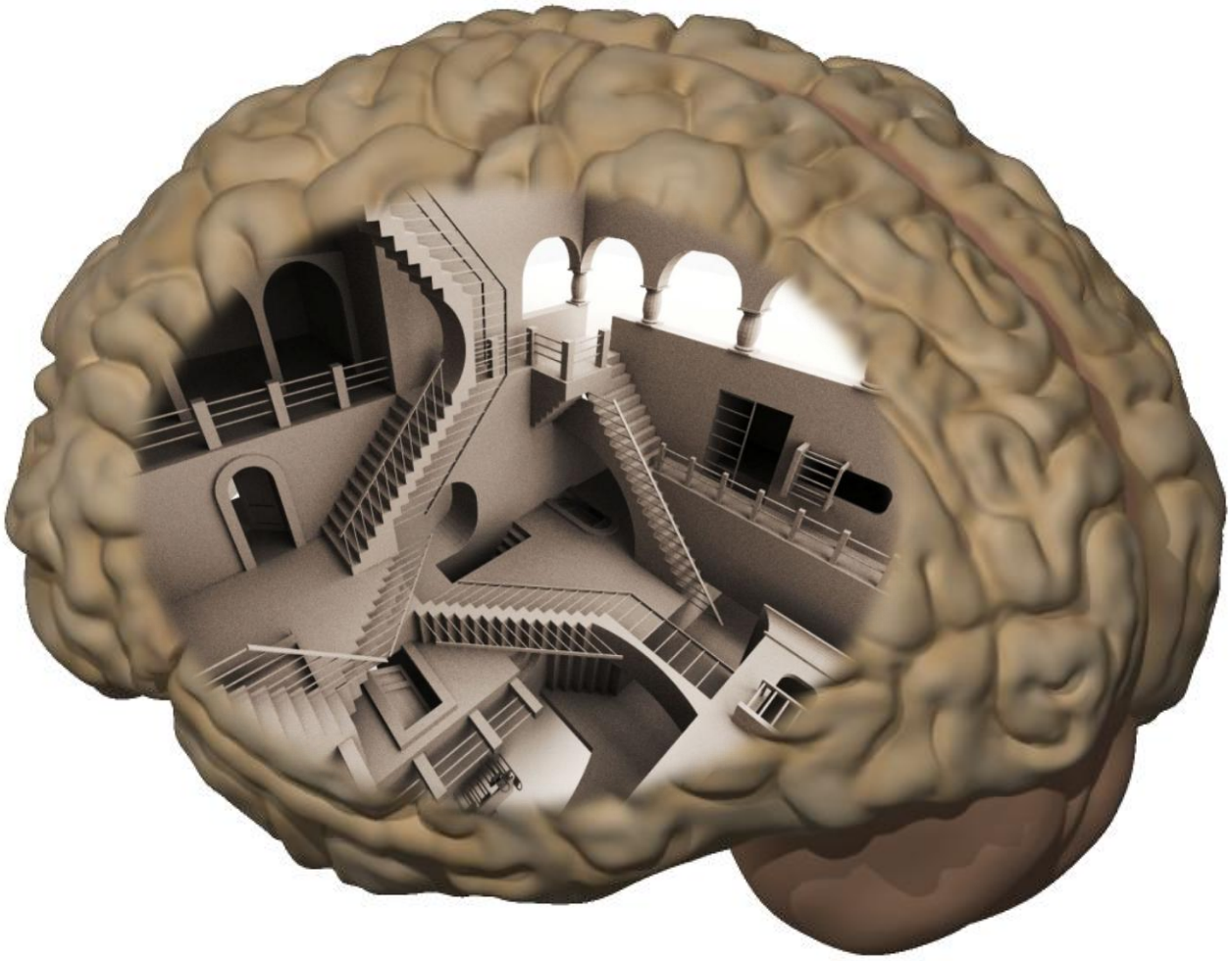
Topological structure of the receptive fields



S. Tanaka 1995
N. Swindale, 1996

Why so much topology?
How is it used???

Neuroscience and geometry



Neuroscience and geometry

The ideal world of geometry

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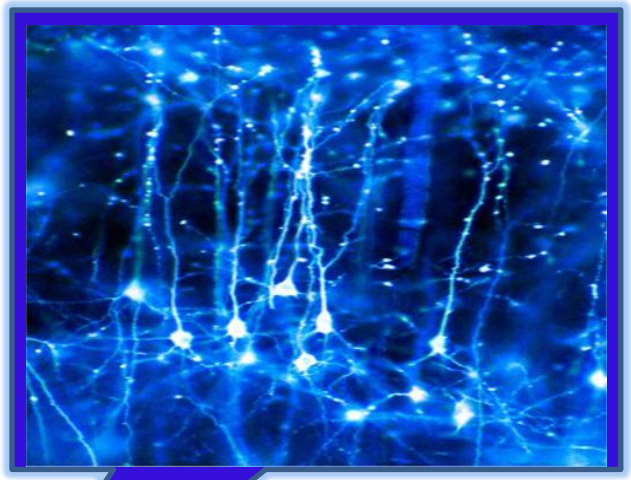

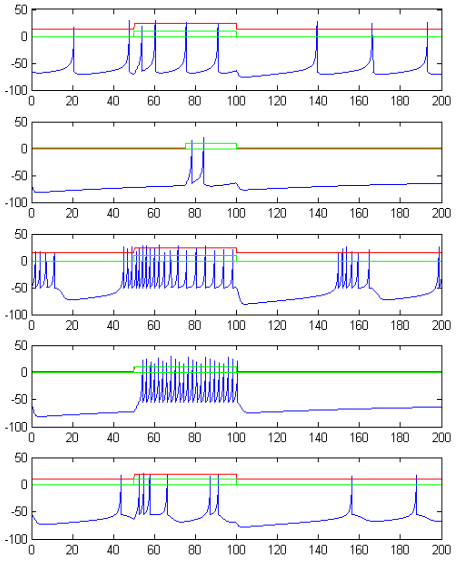

Time code machine

Neuroscience and geometry

…Quapropter bono christiano, sive mathematici, sive quilibet impie divinantium, maxime dicentes vera, cavendi sunt, ne consortio daemoniorum animam deceptam, pacto quodam societatis irretiant.

St. Augustine, Confessions II, xvii, 37

…Hence, a devout Christian must avoid mathematicians and all impious soothsayers, especially when they tell the truth, for fear of leading his soul into error by consorting with demons and entangling himself with the bonds of such association.

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